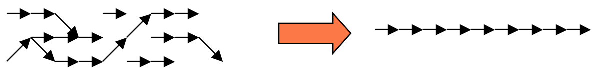
Grouping

To make sense of image elements, they first have to be grouped into larger structures.

Example: Grouping noisy edge elements into a straight edge

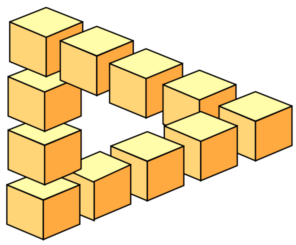


Essential problem:

Obtaining globally valid results by local decisions

Important methods:

- Fitting
- Clustering
- Hough Transform
- Relaxation

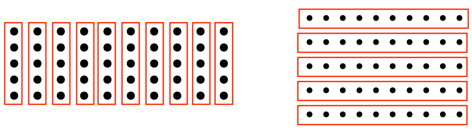


- locally compatible
- globally incompatible

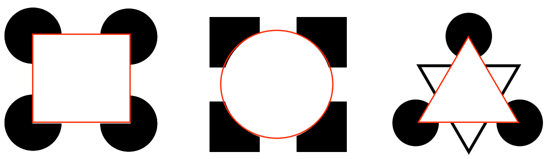
1

Cognitive Grouping

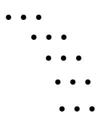
The human cognitive system shows remarkable grouping capabilities



grouping into rows or columns according to a distance criterion



grouping into virtual edges



grouping into virtual motion

It is worthwhile wondering which cognitive grouping rules should also be followed by machine vision

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Fitting Straight Lines

Why do we want to discover straight edges or lines in images?

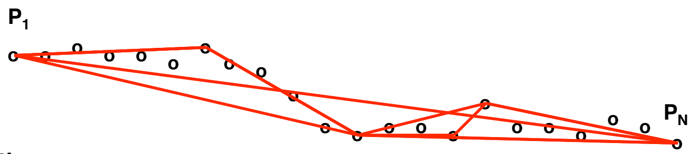
- **Straight edges occur abundantly in the civilized world.**
- **Approximately straight edges are also important to model many natural phenomena, e.g. stems of plants, horizon at a distance.**
- **Straightness in scenes gives rise to straighness in images.**
- **Straightness discovery is an example of constancy detection which is at the heart of grouping (and maybe even interpretation).**

We will treat several methods for fitting straight lines:

- **Iterative refinement**
- **Mean-square minimization**
- **Eigenvector analysis**
- **Hough transform**

Straight Line Fitting by Iterative Refinement

Example: Fitting straight segments to a given object motion trajectory



Algorithm:

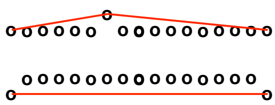
A: First straight line is P_1P_N

B: Is there a straight line segment P_iP_k with an intermediate point P_j $(i < j < k)$ whose distance from P_iP_k is more than d? If no, then terminate.

C: Segment P_iP_k into P_iP_j and P_jP_k and go to B.

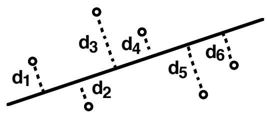
Advantage: simple and fast

Disadvantages:
- **strong effect of outliers**
- **not always optimal**



Straight Line Fitting by Eigenvector Analysis (1)

Given:	$(x_i\ y_i)$ $i = 1 \dots N$
Wanted:	Coefficients c_0, c_1 for straight line $y = c_0 + c_1 x$ which minimizes $\Sigma\, d_i^2$



Observation:

The optimal straight line passes through the mean of the given points. Why?

Let (x´y´) be a coordinate system with the x´ axis parallel to the optimal straight line.

optimal straight line	$x' = x_0'$
error	$\Sigma\, d_i^2 = \Sigma\, (x_i' - x_0')^2$
condition for optimum	$\delta/\delta x_0\ \{\Sigma\, (x_i' - x_0')^2\} = -2 \cdot \Sigma\, (x_i' - x_0') = 0$
	$x_0' = 1/N \cdot \Sigma\, x_i'$

A new coordinate system may be chosen with the origin at the mean of the given points: $x_j' = x_j - \frac{\sum x_i}{N}$ $\quad y_j' = y_j - \frac{\sum y_i}{N}$

Optimal straight line passes through origin, only direction is unknown.

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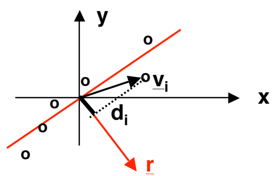
Straight Line Fitting by Eigenvector Analysis (2)

After coordinate transformation the new problem is:

Given:	points $\underline{v}_i^T = [x_i\ y_i]$ with $\Sigma\, \underline{v}_i = \underline{0}$ $i = 1 \dots N$
Wanted:	direction vector $\underline{r}$ which minimizes $\Sigma\, d_i^2$



Minimize $\quad d^2 = \sum_{i=1}^{N} d_i^2 = \sum_{i=1}^{N} (\underline{r}^T \underline{v}_i)^2 = \sum_{i=1}^{N} (\underline{r}^T \underline{v}_i)(\underline{v}_i^T \underline{r}) = \underline{r}^T S \underline{r}$

↑ scatter matrix

Minimization with Lagrange multiplier λ:

$\underline{r}^T S \underline{r} + \lambda \underline{r}^T \underline{r}$ => minimum subject to $\underline{r}^T \underline{r} = 1$

Minimizing $\underline{r}$ is eigenvector of S, minimum is eigenvalue of S.

For a 2D scatter matrix there exist 2 orthogonal eigenvectors:

$\underline{r}_{min}$	orthogonal to optimal straight line
$\underline{r}_{max}$	parallel to optimal straight line

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Straight Line Fitting by Eigenvector Analysis (3)

<u>Computational procedure</u>:

- Determine mean $\underline{m}$ of given points with $m_x = 1/N\ \Sigma\ x_i$, $m_y = 1/N\ \Sigma\ y_i$, i = 1 ... N
- Determine scatter matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \Sigma (x_i-m_x)^2 & \Sigma (x_i-m_x)(y_i-m_y) \\ \Sigma (x_i-m_x)(y_i-m_y) & \Sigma (y_i-m_y)^2 \end{bmatrix}$
- Determine maximal eigenvalue

$$\lambda_{1,2} = \frac{S_{11}+S_{22}}{2} \pm \sqrt{\left(\frac{S_{11}+S_{22}}{2}\right)^2 - |S|} \qquad \lambda_{max} = \max\ \{\lambda_1, \lambda_2\}$$

- Determine direction of eigenvector corresponding to λ_{max}

$S_{11}\ r_x + S_{12}\ r_y = \lambda_{max}\ r_x$ by definition of eigenvector => r_y/r_x

- Determine optimal straight line

$(y- m_y) = (x-m_x)\ (r_y/r_x) = (x-m_x)\ (\lambda_{max} - S_{11})/S_{12}$

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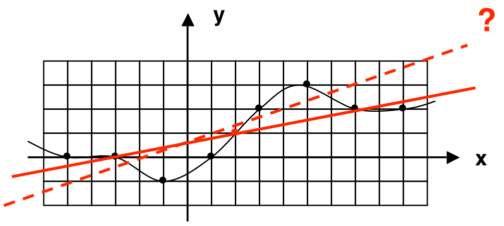
Example for Straight Line Fitting by Eigenvector Analysis

What is the best straight-line approximation of the contour?



Given points: { (-5 0) (-3 0) (-1 -1) (1 0) (3 2) (5 3) (7 2) (9 2) }

Center of gravity: $m_x = 2$ $m_y = 1$

Scatter matrix: $S_{11} = 168$ $S_{12} = S_{21} = 38$ $S_{22} = 14$

Eigenvalues: $\lambda_1 = 176{,}87$ $\lambda_2 = 5{,}13$

Direction of straight line: $r_y/r_x = 0{,}23$

Straight line equation: y = 0,23 x + 0,54

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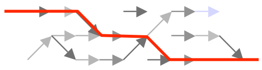
Grouping by Search



What is the "best path" which could represent a boundary in a given field of edgels?

The problem can be formulated as a search problem:

What is the best path from a starting point to an end point, given a cost function $c(x_1, x_2, \dots, x_N)$?

The variables $x_1 \dots x_N$ are decision variables whose values determine the path.

Unfortunately, the total cost $c(x_1, \dots, x_N)$ is in general not minimized by local minimal cost decisions $\min c(x_i)$, e.g. following the path of maximal edgel strength.

Hence search for a global optimum is necessary, e.g.

- hill climbing
- A* search
- **Dynamic Programming**

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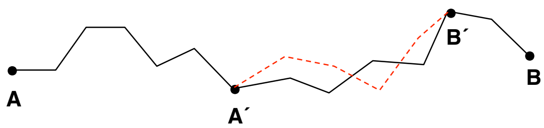
Dynamic Programming (1)

Dynamic Programming is an optimization method which can be applied if the global cost $c(x_1, x_2, \dots, x_N)$ obeys the principle of optimality:

> If $a_1, a_2, \dots, a_N$ minimize $c(x_1, x_2, \dots, x_N)$,
> then $a_i, a_{i+1}, \dots, a_k$ minimize $c(a_i, x_{i+1}, x_{i+2}, \dots, x_{k-1}, a_k)$

Hence, for a globally optimal path every subpath has to be optimal.

Example: In street traffic, an optimal path from A to B usually implies that all subpaths from A´ to B´ between A and B are also optimal.



Dynamic Programming avoids cost computations for all value assignments for $x_1, x_2, \dots, x_N$.
If each x_i, $i = 1 \dots N$, has K possible values, only $N{*}K^2$ cost computations are required instead of K^N.

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Dynamic Programming (2)

Suppose $c(x_1, x_2, \ldots, x_N) = c(x_1, x_2) + c(x_2, x_3) + \ldots + c(x_{N-1}, x_N)$, then the optimality principle holds.

Dynamic Programming:

Step 1: Minimize $c(x_1, x_2)$ over x_1 => $f_1(x2)$
Step 2: Minimize $f_1(x2)+c(x_2, x_3)$ over x_2 => $f_2(x3)$
Step 3: Minimize $f_2(x3)+c(x_3, x_4)$ over x_3 => $f_3(x4)$
•
•
•
Step N: Minimize $f_{N-1}(x_N)$ over x_N => $f_N = \min c(x_1, x_2, \ldots, x_N)$

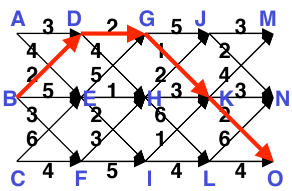
Example of a cost function for boundary search:
"Punish accumulated curvature and reward accumulated edge strengths"

$$c(x_1,\ldots,x_N) = \sum_{k=1\ldots N}(1 - s(x_k)) + \alpha \sum_{k=1\ldots N-1} q(x_k, x_{k+1})$$

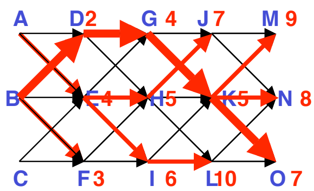
$s(x_k)$ edge strength
$q(x_k, x_{k+1})$ curvature

Dynamic Programming (3)

<u>Example</u>: Find optimal path from left to right



optimaler Pfad?



optimaler Pfad!

- Find best paths from A, B, C to D, E, F, record optimal costs at D, E, F
- Find best paths from D, E, F to G, H, I, record optimal costs at G, H, I

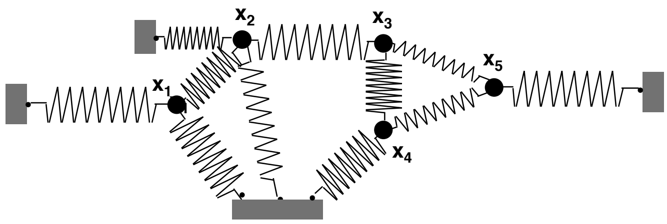
etc.

- Trace back optimal path from right to left

Grouping by Relaxation

Relaxation methods seek a solution by stepwise minimization ("relaxation") of constraints.

Analogy with spring system:



Variables x_i take on values (= positions) where springs are maximally relaxed corresponding to a state of global minimal energy. Hence relaxation is often realized by "energy minimization".

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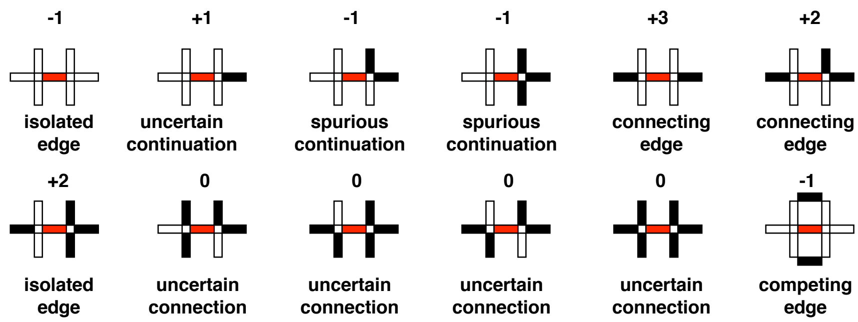
Contexts for Edge Relaxation

Iterative modification of edge strengths using context-dependent compatibility rules.

Context types:



Each context contributes with weight $w_j = w_0 \cdot \{-1 \dots +2\}$ to an interative modification of the edge strength of the central element.

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Modification Rule for Edge Relaxation

P_i^k edge strength in position i after iteration k
Q_{ij}^k strength of context j for position i after iteration k
w_j weight factor of context j

$Q_{ij}^k = \prod P_m^k \cdot \prod (1-P_n^k)$ edge context strength

m, n ranging over all supporting and not supporting edge positions of context j, respectively.

$$P_i^{k+1} = P_i^k \frac{1+\Delta P_i^k}{1+P_i^k \Delta P_i^k}$$ edge strength modification rule

$$\Delta P_i^k = \sum_{j=1}^{N} w_j Q_{ij}^k$$ edge strength increment

There is empirical evidence (but no proof) that for most edge images this relaxation procedure converges within 10 ... 20 iterations.

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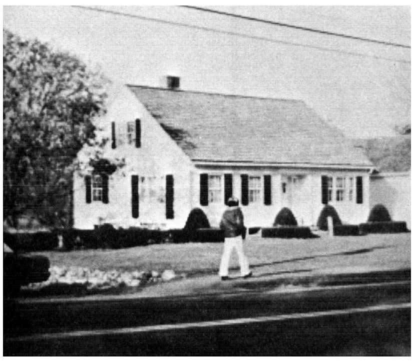
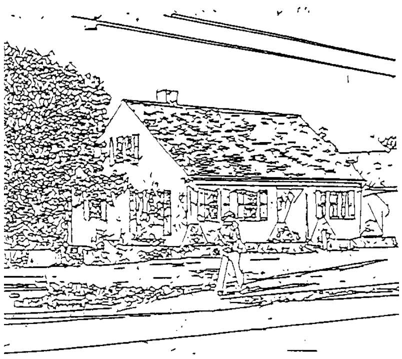
Example of Edge-finding by Relaxation





Landhouse scene from VISIONS project, 1982

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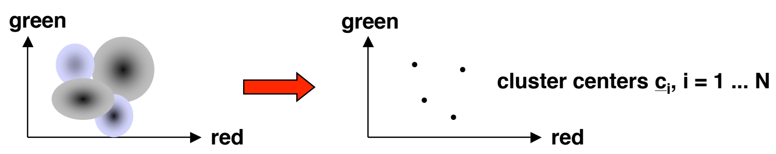
Histogram-based Segmentation with Relaxation (1)

<u>Basic idea</u>:

Use relaxation to introduce a local similarity constraint into histogram-based region segmentation.

A Determine cluster centers by multi-dimensional histogram analysis



B Label each pixel by cluster-membership probabilities p_i , 1 = 1 ... N

$$p_i = \frac{1/d_i}{\sum_{k=1}^{N} 1/d_k}$$

d_i is Euclidean distance between the feature vector of the pixel and cluster center c_i

Histogram-based Labelling with Relaxation (2)

C Iterative relaxation of the $p_i(j)$ of all pixels j:

- equal labels of neighbouring pixels support each other
- unequal labels of neighbouring pixels inhibit each other

$$q_i(j) = \sum_{k \in D(j)} [w^+ p_i(k) - w^- (1 - p_i(k))]$$

D(j) is neighbourhood of pixel j

$$p_i'(j) = \frac{p_i(j) + q_i(j)}{\sum_n (p_n(j) + q_n(j))}$$

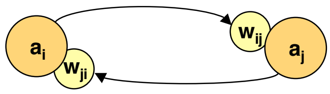
new probability $p_i'(j)$ of pixel j to belong to cluster i

D Region assignment of each pixel according to its maximal membership probability max p_i

E Recursive application of the procedure to individual regions

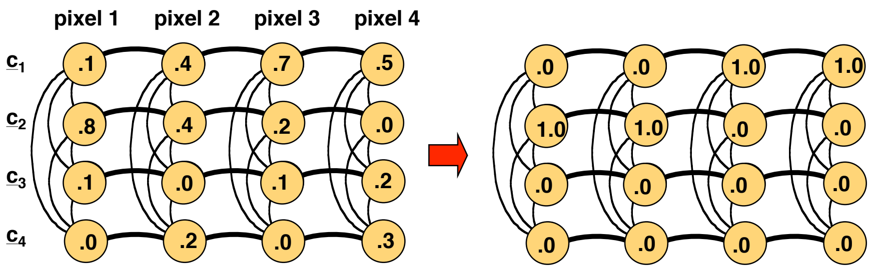
Relaxation with a Neural Network

Principle:



cells influence each other´s activation via exciting or inhibiting weights

Relaxation labelling of 4 pixels:



bidirectional inhibiting connection

bidirectional exciting connection

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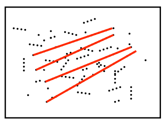
Hough Transform (1)

Robust method for fitting straight lines, circles or other geometric figures which can be described analytically.

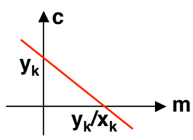
Given: Edge points in an image

Wanted: Straight lines supported by the edge points



An edge point (x_k, y_k) supports all straight lines $y = mx + c$ with parameters m and c such that $y_k = mx_k + c$.

The locus of the parameter combinations for straight lines through (x_k, y_k) is a straight line in parameter space.



Principle of Hough transform for straight line fitting:

- Provide accumulator array for quantized straight line parameter combinations
- For each edge point, increase accumulator cells for all parameter combinations supported by the edge point
- Maxima in accumulator array correspond to straight lines in the image

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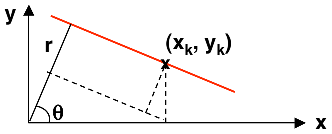
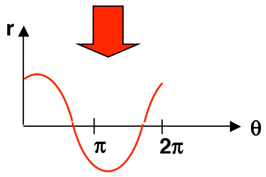
Hough Transform (2)

For straight line finding, the parameter pair (r, θ) is commonly used because it avoids infinite parameter values:

$$x_k\cos\theta + y_k\sin\theta = r$$



Each edge point (x_k, y_k) corresponds to a sinusoidal in parameter space:



Important improvement by exploiting direction information at edge points:

(x_k, y_k, φ) ⇒ $x_k\cos\theta + y_k\sin\theta = r$ restricted to $\varphi-\delta \le \theta \le \varphi+\delta$

φ: gradient direction; δ: direction tolerance

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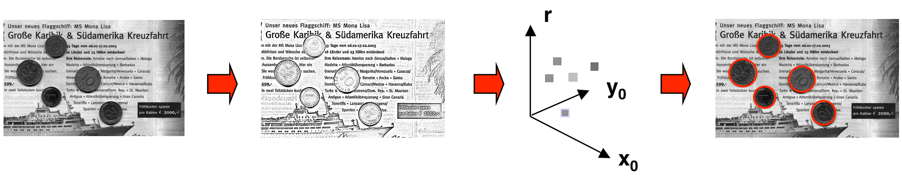
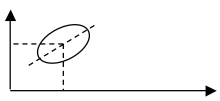
Hough Transform (3)

Same method may be applied to other parameterizable shapes, e.g.

- circles $(x_k-x_0)^2 + (y_k-y_0)^2 = r^2$ 3 parameters x_0, y_0, r



- ellipses $$\left(\frac{(x_k - x_0)\cos\gamma + (y_k - y_0)\sin\gamma}{a}\right)^2 + \left(\frac{(y_k - y_0)\cos\gamma - (x_k - x_0)\sin\gamma}{b}\right)^2 = 1$$ 5 parameters x_0, y_0, a, b, γ



Accumulator arrays grow exponentially with number of parameters
=> quantization must be chosen with care

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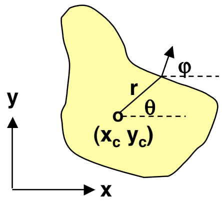
Generalized Hough Transform

- shapes are described by edge elements ($r\ \theta\ \varphi$) relative to an arbitrary reference point ($x_c\ y_c$)
- φ is used as index into ($\rho\ \theta$) pairs of a shape description
- edge point coordinates ($x_k\ y_k$) and gradient direction φ_k determine possible reference point locations
- likely reference point locations are determined via maxima in accumulator array



φ_1: $\{(r_{11}\ \theta_{11})\ (r_{12}\ \theta_{12})\ \ldots\}$

φ_2: $\{(r_{21}\ \theta_{11})\ (r_{22}\ \theta_{12})\ \ldots\}$

$\vdots$

φ_N: $\{(r_{N1}\ \theta_{11})\ (r_{N2}\ \theta_{12})\ \ldots\}$

$(x_k\ y_k\ \varphi_k) \Rightarrow \{(x_c\ y_c)\} = \{\ (x_k - r_i(\varphi_k)\cos\theta_i(\varphi_k)),\ (y_k - r_i(\varphi_k)\sin\theta_i(\varphi_k))\ \}$

↳ counter cell in accumulator array

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