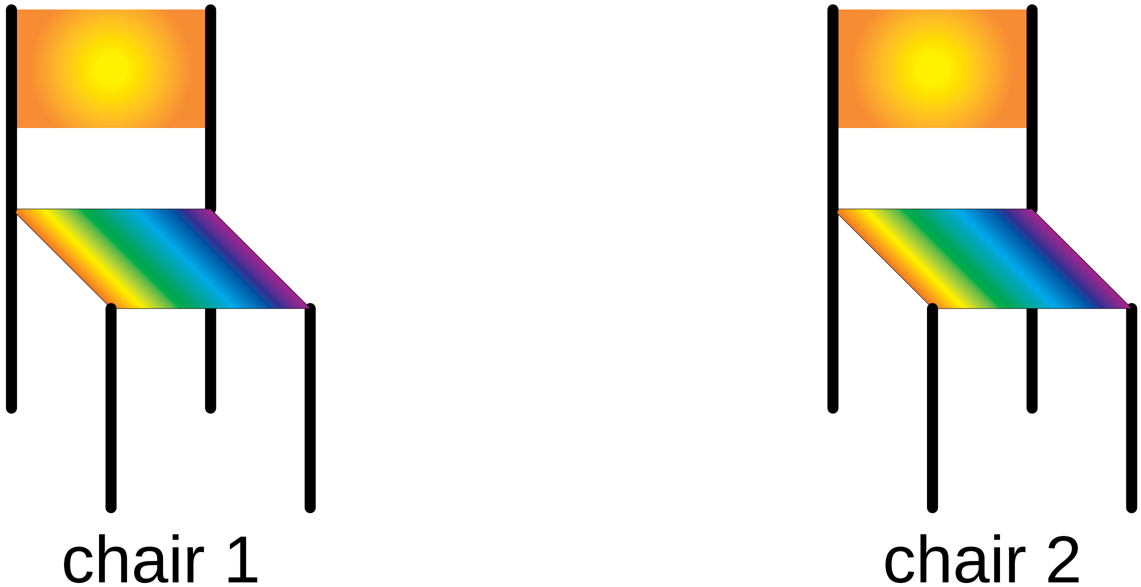
Equality

- Sometimes two terms denote the same individual.
- Example: Clark Kent & superman. 4×4 & $11 + 5$. The projector we used last Friday & this projector.
- Ground term t_1 **equals** ground term t_2, written $t_1 = t_2$, is true in interpretation I if t_1 and t_2 denote the same individual in interpretation I.

Equality doesn't mean similarity



$chair1 \neq chair2$

$chair\_on\_right = chair2$

$chair\_on\_right$ is not similar to $chair2$, it is $chair2$.

Why is equality important?

- In a doctor's office, the doctor wants to know if a patient is the same patient that she saw last week (or is his twin sister).
- In a criminal investigation, the police want to determine if someone is the same person as the person who committed some crime.
- When buying a replacement switch, an electrician may want to know if it was built in the same factory as the switches that were unreliable. (And if it is a different switch to the one that was replaced the previous time).

Allowing Equality Assertions

- Without equality assertions, the only thing that is equal to a ground term is itself.

  This can be captured as though you had the assertion $X = X$. Explicit equality never needs to be used.

- If you allow equality assertions, you need to derive what follows from them. Either:
  - axiomatize equality like any other predicate
  - build special-purpose inference machinery for equality

Axiomatizing Equality

$$X = X.$$

$$X = Y \leftarrow Y = X.$$

$$X = Z \leftarrow X = Y \wedge Y = Z.$$

For each n-ary function symbol f there is a rule of the form

$$f(X_1, \ldots, X_n) = f(Y_1, \ldots, Y_n) \leftarrow$$
$$X_1 = Y_1 \wedge \cdots \wedge X_n = Y_n.$$

For each n-ary predicate symbol p, there is a rule of the form

$$p(X_1, \ldots, X_n) \leftarrow$$
$$p(Y_1, \ldots, Y_n) \wedge X_1 = Y_1 \wedge \cdots \wedge X_n = Y_n.$$

Special-Purpose Equality Reasoning

Paramodulation: If you have $t_1 = t_2$, then you can replace any occurrence of t_1 by t_2.

Treat equality as a **rewrite rule**, substituting equals for equals.

You select a **canonical representation** for each individual and rewrite all other representations into that representation.

Example: treat the sequence of digits as the canonical representation of the number.

Example: use the student number as the canonical representation for students.

Unique Names Assumption

The convention that different ground terms denote different individuals is the **unique names assumption.**

For every pair of distinct ground terms t_1 and t_2, assume $t_1 \neq t_2$, where "$\neq$" means "not equal to."

Example: For each pair of courses, you don't want to have to state, $math302 \neq psyc303$, …

Example: Sometimes the unique names assumption is inappropriate, for example $3 + 7 \neq 2 \times 5$ is wrong.

Axiomatizing Inequality for the UNA

- $c \neq c'$ for any distinct constants c and c'.
- $f(X_1, \ldots, X_n) \neq g(Y_1, \ldots, Y_m)$ for any distinct function symbols f and g.
- $f(X_1, \ldots, X_n) \neq f(Y_1, \ldots, Y_n) \leftarrow X_i \neq Y_i$, for any function symbol f. There are n instances of this schema for every n-ary function symbol f (one for each i such that $1 \leq i \leq n$).
- $f(X_1, \ldots, X_n) \neq c$ for any function symbol f and constant c.
- $t \neq X$ for any term t in which X appears (where t is not the term X).

Top-down procedure and the UNA

- Inequality isn't just another predicate. There are infinitely many answers to $X \neq f(Y)$.
- If you have a subgoal $t_1 \neq t_2$, for terms t_1 and t_2 there are three cases:
  - t_1 and t_2 don't unify. In this case, $t_1 \neq t_2$ succeeds.
  - t_1 and t_2 are identical including having the same variables in the same positions. Here $t_1 \neq t_2$ fails.
  - Otherwise, there are instances of $t_1 \neq t_2$ that succeed and instances of $t_1 \neq t_2$ that fail.

Implementing the UNA

- **Recall:** in SLD resolution you can select any subgoal in the body of an answer clause to solve next.
- **Idea:** only select inequality when it will either succeed or fail, otherwise select another subgoal. Thus you are **delaying** inequality goals.
- If only inequality subgoals remain, and none fail, the query succeeds.

Inequality Example

$$notin(X, []).$$

$$notin(X, [H|T]) \leftarrow X \neq H \wedge notin(X, T).$$

$$good\_course(C) \leftarrow course(C) \wedge passes\_analysis(C).$$

$$course(cs312).$$

$$course(cs444).$$

$$course(cs322).$$

$$passes\_analysis(C) \leftarrow something\_complicated(C).$$

$$?notin(C, [cs312, cs322, cs422, cs310, cs402])$$
$$\wedge good\_course(C).$$