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A *TCSP*[1]-like decidable constraint language generalising existing cardinal direction relations

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Abstract. We define a quantitative constraint language subsuming two calculi well-known in QSR[4]: Frank's cone-shaped and projection-based calculi of cardinal direction relations. The language is based on convex constraints of the form $(\alpha, \beta)(x, y)$, with $\alpha, \beta \in [0, 2\pi)$ and $(\beta - \alpha) \in [0, \pi)$: the meaning of such a constraint is that point x belongs to the (convex) cone-shaped area rooted at y, and bounded by angles α and β. The general form of a constraint is a disjunction of the form $[(\alpha_1, \beta_1) \vee \cdots \vee (\alpha_n, \beta_n)](x, y)$, with $(\alpha_i, \beta_i)(x, y)$, $i = 1 \ldots n$, being a convex constraint as described above: the meaning of such a general constraint is that, for some $i = 1 \ldots n$, $(\alpha_i, \beta_i)(x, y)$ holds. A conjunction of such general constraints is a *TCSP*-like CSP, which we will refer to as an *SCSP* (Spatial Constraint Satisfaction Problem). We describe how to compute converse, intersection and composition of *SCSP* constraints, allowing thus to achieve path consistency for an *SCSP*. We show how to translate a convex constraint into a conjunction of linear inequalities on variables consisting of the arguments' coordinates. Our approach to effectively solving a general *SCSP* is then to adopt a solution search algorithm using (1) path consistency as the filtering method during the search, and (2) the Simplex algorithm, guaranteeing completeness, at the leaves of the search tree.

Keywords: Constraint Satisfaction, Spatial reasoning, Geometric Reasoning, Knowledge Representation, Qualitative Reasoning, Quantitative Reasoning

1 Introduction

Conciliating qualitative reasoning and quantitative reasoning in KR&R systems: a way to systems representationally more flexible, cognitively more plausible, and, computationally, with the advantage of having the choice between a purely-quantitative and a qualitative-computations-first behaviours.

Knowledge representation (KR) systems allowing for the representation of both qualitative knowledge and quantitative knowledge are more than needed by modern applications (see, e.g., [2]), which, depending on the level of detail of the knowledge to be represented, may feel happy with a high-level, qualitative language, or need to use a low-level, quantitative language. Qualitative languages suffer from what Forbus et al. [7] refer to as the poverty conjecture, but have the advantage of behaving computationally better. On the other hand, quantitative languages do not suffer from the poverty conjecture, but have a slow computatinal behaviour. Thus, such a KR system will feel happier when the knowledge at hand can be represented in a purely qualitative way, for it can then get rid of heavy numeric calculations, and restrict its computations to a manipulation of symbols, consisting, in the case of constraint-based languages in the style of the Region-Connection Calculus RCC-8 [15], mainly in computing a closure under a composition table.

An important question raised by the above discussion is clearly how to augment the chances of a qualitative/quantitative KR system to remain at the qualitative level. Consider, for instance, QSR constraint-based, RCC-8-like languages. Given the poverty conjecture, which corresponds to the fact that such a language can make only a finite number of distinctions, reflected by the number of its atomic relations, one way of answering the question could be to integrate more than one QSR language within the same KR system. The knowledge at hand is then handled in a quantitative way only in the extreme case when it can be represented by none of the QSR languages which the system integrates.

One way for a KR system, such as described above, to reason about its knowledge is to start with reasoning about the qualitative part of the knowledge, which decomposes, say, into n components, one for each of the QSR languages the system integrates. For RCC-8-like languages, this can be done using a constraint propagation algorithm such as the one in [1]. If in either of the n components, an inconsistency has been detected, then the whole knowledge has been detected to be inconsistent without the need of going into low-level details. If no inconsistency has been detected at the high, qualitative level, then the whole knowledge needs translation into the unifying quantitative language, and be processed in a purely quantitative way. But even when the high-level, qualitative computations fail to detect any inconsistency, they still potentially help the task of the low-level, purely quantitative computations. The situation can be compared to standard search algorithms in CSPs, where a local-consistency preprocessing is applied to the whole knowledge to potentially reduce the search space, and eventually detect the knowledge inconsistency, before the actual search for a solution starts.

With the above considerations in mind, we consider the integration of Frank's cone-shaped and projection-based calculi of cardinal direction relations [8], well-known in QSR. A complete decision procedure for the projection-based calculus is known from Ligozat's

[1] *TCSPs* stands for Temporal Constraint Satisfaction Problems, a well-known constraint-based temporal framework [6].
[2] European Conference on Artificial Intelligence.
[3] The reviews are added to the actual paper, after the references, for potential people interested in objectivity of conferences' reviewing processes.
[4] Qualitative Spatial Reasoning.

work [12]. For the other calculus, based on a uniform 8-sector partition of the plane, making it more flexible and cognitively more plausible, no such procedure is known. For each of the two calculi, the region of the plane associated with each of the atomic relations is convex, and given by the intersection of two half-planes. As a consequence, each such relation can be equivalently written as a conjunction of linear inequalities on variables consisting of the coordinates of the relation's arguments. We consider a more general, qualitative/quantitative language, which, at the basic level, expresses convex constraints of the form $r(x, y)$, where r is a cone-shaped or projection-based atomic relation of cardinal directions, or of the form $(\alpha, \beta)(x, y)$, with $\alpha, \beta \in [0, 2\pi)$ and $(\beta - \alpha) \in [0, \pi)$: the meaning of $(\alpha, \beta)(x, y)$, in particular, is that point x belongs to the (convex) cone-shaped area rooted at y, and bounded by angles α and β. We refer to such constraints as basic constraints: qualitative basic constraint in the former case, and quantitative basic constraint in the latter. A conjunction of basic constraints can be solved by first applying constraint propagation, based on a composition operation to be defined, which is basically the spatial counterpart of composition of two TCSP constraints [6]. If the propagation detects no inconsisteny then the knowledge is translated into a system of linear inequalities, and solved with the well-known Simplex algorithm. The preprocessing of the qualitative component of the knowledge can be done with a constraint propagation algorithm such as the one in [1], and needs the composition tables of the cardinal direction calculi, which can be found in [8].

The general form of a constraint is $(s_1 \vee \cdots \vee s_n)(x, y)$, which we also represent as $\{s_1, \ldots, s_n\}(x, y)$, where $s_i(x, y)$, for all $i \in \{1, \ldots, n\}$, is a basic constraint, either qualitative or quantitative. The meaning of such a general constraint is that, either of the n basic constraints is satisfied, i.e., $s_1(x, y) \vee \cdots \vee s_n(x, y)$. A general constraint is qualitative if it is the disjunction of qualitative basic constraints of one type, cone-shaped or projection-based; it is quantitative otherwise. The language can be looked at as the spatial counterpart of Dechter et al.'s TCSPs [6]: the domain of a TCSP variable is $\mathbb{R}$, symbolising continuous time, whereas the domain of an SCSP variable is the cross product $\mathbb{R} \times \mathbb{R}$, symbolising the continuous 2-dimensional space.

Due to space limitations, we restrict the presentation to the unifying *TCSP*-like constraint language, will all the tools required to convince the reader that the work is the description of an implementable KR&R system, consisting in a search algorithm using path consistency as the filtering procedure during the search, and the Simplex algorithm as a completeness guarantee, at the leaves of the search space.

2 Constraint satisfaction problems

A constraint satisfaction problem (CSP) of order n consists of:

1. a finite set of n variables, $x_1, \ldots, x_n$;
2. a set U (called the universe of the problem); and
3. a set of constraints on values from U which may be assigned to the variables.

An m-ary constraint is of the form $R(x_{i_1}, \cdots, x_{i_m})$, and asserts that the values $a_{i_1}, \ldots, a_{i_m}$ assigned to the variables $x_{i_1}, \ldots, x_{i_m}$, respectively, are so that the m-tuple $(a_{i_1}, \ldots, a_{i_m})$ belongs to the m-ary relation R (an m-ary relation over the universe U is any subset of U^m). An m-ary CSP is one of which the constraints are m-ary constraints. We will be concerned exclusively with binary CSPs.

For any two binary relations R and S, $R \cap S$ is the intersection of R and S, $R \cup S$ is the union of R and S, $R \circ S$ is the composition of R and S, and $R^{\smile}$ is the converse of R; these are defined as follows:

$$\begin{array}{lcl} R \cap S & = & \{(a, b) : (a, b) \in R \text{ and } (a, b) \in S\}, \\ R \cup S & = & \{(a, b) : (a, b) \in R \text{ or } (a, b) \in S\}, \\ R \circ S & = & \{(a, b) : \text{for some } c, (a, c) \in R \text{ and } (c, b) \in S\}, \\ R^{\smile} & = & \{(a, b) : (b, a) \in R\}. \end{array}$$

Three special binary relations over a universe U are the empty relation $\emptyset$ which contains no pairs at all, the identity relation $\mathcal{I}_U^b = \{(a, a) : a \in U\}$, and the universal relation $\top_U^b = U \times U$.

2.1 Constraint matrices

A binary constraint matrix of order n over U is an $n \times n$-matrix, say $\mathcal{B}$, of binary relations over U verifying the following:

$$\begin{array}{ll} (\forall i \leq n)(\mathcal{B}_{ii} \subseteq \mathcal{I}_U^b) & \text{(the diagonal property)}, \\ (\forall i, j \leq n)(\mathcal{B}_{ij} = (\mathcal{B}_{ji})^{\smile}) & \text{(the converse property)}. \end{array}$$

A binary CSP P of order n over a universe U can be associated with the following binary constraint matrix, denoted $\mathcal{B}^P$:

1. Initialise all entries to the universal relation: $(\forall i, j \leq n)((\mathcal{B}^P)_{ij} \leftarrow \top_U^b)$
2. Initialise the diagonal elements to the identity relation: $(\forall i \leq n)((\mathcal{B}^P)_{ii} \leftarrow \mathcal{I}_U^b)$
3. For all pairs (x_i, x_j) of variables on which a constraint $(x_i, x_j) \in R$ is specified: $(\mathcal{B}^P)_{ij} \leftarrow (\mathcal{B}^P)_{ij} \cap R, (\mathcal{B}^P)_{ji} \leftarrow ((\mathcal{B}^P)_{ij})^{\smile}$.

2.2 Strong k-consistency, refinement

Let P be a CSP of order n, V its set of variables and U its universe. An instantiation of P is any n-tuple $(a_1, a_2, \ldots, a_n)$ of U^n, representing an assignment of a value to each variable. A consistent instantiation is an instantiation $(a_1, a_2, \ldots, a_n)$ which is a solution: $(\forall i, j \leq n)((a_i, a_j) \in (\mathcal{B}^P)_{ij})$. P is consistent if it has at least one solution; it is inconsistent otherwise. The consistency problem of P is the problem of verifying whether P is consistent.

Let $V' = \{x_{i_1}, \ldots, x_{i_j}\}$ be a subset of V. The sub-CSP of P generated by V', denoted $P_{|V'}$, is the CSP with V' as the set of variables, and whose constraint matrix is obtained by projecting the constraint matrix of P onto V': $(\forall k, l \leq j)((\mathcal{B}^{P_{|V'}})_{kl} = (\mathcal{B}^P)_{i_k i_l})$. P is k-consistent [9, 10] (see also [4]) if for any subset V' of V containing $k-1$ variables, and for any variable $X \in V$, every solution to $P_{|V'}$ can be extended to a solution to $P_{|V' \cup \{X\}}$. P is strongly k-consistent if it is j-consistent, for all $j \leq k$.

1-consistency, 2-consistency and 3-consistency correspond to node-consistency, arc-consistency and path-consistency, respectively [13, 14]. Strong n-consistency of P corresponds to what is called global consistency in [5]. Global consistency facilitates the important task of searching for a solution, which can be done, when the property is met, without backtracking [10].

A refinement of P is a CSP P' with the same set of variables, and such that: $(\forall i, j)((\mathcal{B}^{P'})_{ij} \subseteq (\mathcal{B}^P)_{ij})$.

3 A spatial counterpart of *TCSPs*: Spatial Constraint Satisfaction Problems (*SCSPs*)

TCSPs (Temporal Constraint Satisfaction Problems) is a constraint-based framewrok well-known in Temporal Reasoning [6]. We provide a spatial counterpart of *TCSPs*, which we refer to as *SCSPs* — Spatial Constraint Satisfaction Problems. The domain of an *SCSP* variable is the cross product $\mathbb{R} \times \mathbb{R}$, which we look at as the set of points of the 2-dimensional space. As for a *TCSP*, an *SCSP* will have unary constraints and binary constraints, and unary constraints can be interpreted as special binary constraints by choosing an origin of the 2-dimensional space —space $(0, 0)$.

We first define some more terminology to be used in the rest of the paper. We make use of a Cartesian system of coordinates $(O, x'x, y'y)$. The x-axis $x'x$ is the origin of angles, and the anticlockwise orientation is the positive orientation for angles. Given that we use the set $[0, 2\pi)$ as the universe of angles (measured in radians), if two angles α and β are so that $\alpha > \beta$, the interval $\langle^{\imath}\alpha, \beta\rangle^{\jmath}$ will represent the union $\langle^{\imath}\alpha, 2\pi) \cup [0, \beta\rangle^{\jmath}$. Given a positive real number α and a strictly positive integer n, we denote by α *mod* n the remainder of the integral division of α by n. Furthermore, given any $\alpha, \beta \in [0, 2\pi)$, the difference $\beta \ominus \alpha$ will measure the anticlockwise (angular) distance of β relative to α: i.e., $\beta \ominus \alpha = (\frac{\beta - \alpha + 2\pi}{\pi} \textit{ mod } 2)\pi$; similarly, the sum $\alpha \oplus \beta$ of α and β is defined as $\alpha \oplus \beta = (\frac{\alpha + \beta}{\pi} \textit{ mod } 2)\pi$.

Definition 1 (*SCSP*) *An* SCSP *consists of (1) a finite number of variables ranging over the universe of points of the 2-dimensional space (henceforth 2D-points); and (2)* SCSP *constraints on the variables.*

An *SCSP* constraint is either unary or binary, and either basic or disjunctive. A basic constraint is (1) of the form $e(x, y)$, e being equality, or (2) of the general form $\langle^{\imath}\alpha, \beta\rangle^{\jmath}(x, y)$ (binary) or $\langle^{\imath}\alpha, \beta\rangle^{\jmath}(x)$ (unary), with $\alpha, \beta \in [0, 2\pi)$, $(\beta \ominus \alpha) \in [0, \pi)$, $\imath, \jmath \in \{0, 1\}$. '$\langle^0$' and '$\langle^1$' stand, respectively, for the left open bracket '(' and the left close bracket '['. Similarly, '$\rangle^0$' and '$\rangle^1$' stand, respectively, for the right open bracket ')' and the right close bracket ']'. A graphical illustration of a general basic constraint is provided in Figure 1.

A disjunctive constraint is of the form $[S_1 \vee \cdots \vee S_n](x, y)$ (binary) or $[S_1 \vee \cdots \vee S_n](x)$ (unary), with $S_k(x, y)$ and $S_k(x)$, $k = 1 \ldots n$, being basic constraints as described above: in the binary case, the meaning of such a disjunctive constraint is that, for some $k = 1 \ldots n$, $S_k(x, y)$ holds; similarly, in the unary case, the meaning is that, for some $k = 1 \ldots n$, $S_k(x)$ holds. A unary constraint $R(x)$ may be seen as a special binary constraint if we consider an origin of the World (space $(0, 0)$), represented, say, by a variable x_0: $R(x)$ is then equivalent to $R(x, x_0)$. Unless explicitly stated otherwise, we assume, in the rest of the paper, that the constraints of an *SCSP* are all binary.

An *SCSP* constraint $R(x, y)$ is convex if, given an instantiation $y = a$ of y, the set of points x satisfying $R(x, a)$ is a convex subset of the plane. A universal *SCSP* constraint is an *SCSP* constraint of the form $[0, 2\pi)(x, y)$: the knowledge consisting of such a constraint is equivalent to "no knowledge", i.e., any instantiation (a, b) of the pair (x, y) satisfies it. A universal constraint is also a convex constraint. A convex *SCSP* is an *SCSP* of which all the constraints are convex. Given its similarity with an *STP* (Simple Temporal Problem) [6], we refer to a convex *SCSP* as an *SSP* (Simple Spatial Problem). An *SCSP* is basic if all its constraints are basic. We refer to a basic *SCSP* as a *BSP* (Basic Spatial Problem). Note that a *BSP* may have pairs (x, y) of variables on which no constraint is specified (the implicit constraint on such pairs is then the universal relation $[0, \pi)$, which we also refer to as ?).

The standard path consistency procedure for binary CSPs is guided by three algebraic operations, the converse of a constraint, the composition of two constraints, and the intersection of two constraints. These are defined below for *SCSP* basic constraints. The case of general (possibly disjunctive) constraints is obtainable from the case of basic constraints.

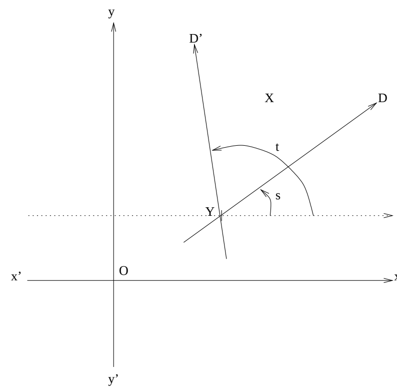

Figure 1. Graphical interpretation of the basic constraint $\langle^{\imath}s, t\rangle^{\jmath}(X, Y)$: Given Y, the set of points X satisfying the constraint $\langle^{\imath}s, t\rangle^{\jmath}(X, Y)$ is the cone-shaped area centred at Y, whose lower bound (open if $\imath = 0$, close otherwise) and upper bound (open if $\jmath = 0$, close otherwise) are, respectively, the half-lines whose angular distances from the x-axis, with respect to anticlockwise orientation, are s and t.

3.1 The converse of an *SCSP* basic constraint

The converse of an *SCSP* relation R is the *SCSP* relation $R^{\smile}$ such that, for all x, y, $R(x, y)$ *iff* $R^{\smile}(y, x)$. We refer to the constraint $R^{\smile}(y, x)$ as the converse of the constraint $R(x, y)$. The converse of $e(x, y)$ is clearly $e(y, x)$. The converse of an *SCSP* basic constraint $\langle^{\imath}\alpha, \beta\rangle^{\jmath}(x, y)$ is the *SCSP* basic constraint $\langle^{\imath}\alpha \oplus \pi, \beta \oplus \pi\rangle^{\jmath}(y, x)$, which can be explained by the simple fact that, given any instantiation $(x, y) = (a, b)$ of the pair (x, y) satisfying the constraint $\langle^{\imath}\alpha, \beta\rangle^{\jmath}(x, y)$, the angle formed by the directed line (ba) with the x-axis is obtained by adding π to the angle formed by the directed line (ab) with the x-axis.

3.2 The composition of two *SCSP* basic constraints

Consider a point y of the plane, and an angle α in $[0, \pi)$. We denote by $l(y, \alpha)$ the directed line through y forming angle α with the x-axis $x'x$. y and α partition the plane into five zones, which are the left open half-plane bounded by $l(y, \alpha)$, the half-line consisting of the points of $l(y, \alpha)$ coming before y (negative half-line), the point y itself, the half-line consisting of the points of $l(y, \alpha)$ coming after y (positive half-line), and the right open hal-plane bounded by $l(y, \alpha)$. We denote the five regions by *lohp* (y, α), *nhl* (y, α), *pt-reg* (y, α), *phl* (y, α), and *rohp* (y, α), respectively, and the set of all of them by *REGIONS* (y, α). Given a fixed angle α in $[0, \pi)$, we can thus define a five-atom calculus $\mathcal{CAL}_\alpha$ of binary relations. The atoms are $\textit{lohp}_\alpha$, $\textit{nhl}_\alpha$, *EQ*, $\textit{phl}_\alpha$ and $\textit{rohp}_\alpha$, defined as follows, for all pairs (x, y) of 2D points: $\textit{lohp}_\alpha(x, y)$ *iff* x belongs to *lohp* (y, α), $\textit{nhl}_\alpha(x, y)$ *iff* x belongs to *nhl* (y, α), $\textit{EQ}(x, y)$ *iff* $x = y$, $\textit{phl}_\alpha(x, y)$ *iff* x belongs to *phl* (y, α), and $\textit{rohp}_\alpha(x, y)$ *iff* x belongs to *rohp* (y, α). We denote by $\textit{ATOMS}(\alpha)$ the set of all five atoms. Clearly, $\textit{lohp}_\alpha(x, y)$ *iff* $\textit{rohp}_\alpha(y, x)$, $\textit{nhl}_\alpha(x, y)$ *iff* $\textit{phl}_\alpha(y, x)$, and $\textit{EQ}(x, y)$ *iff* $\textit{EQ}(y, x)$. In other words, $\textit{lohp}_\alpha$ and $\textit{rohp}_\alpha$ are each other's converses, and so are $\textit{nhl}_\alpha$ and $\textit{phl}_\alpha$; whereas *EQ* is its own converse. We consider now two fixed angles α and β from $[0, \pi)$ and compute the composition $R_1 \circ R_2$ of R_1 and R_2, with $R_1 \in \textit{ATOMS}(\alpha)$ and $R_2 \in \textit{ATOMS}(\beta)$. $R_1 \circ R_2$ is the relation $R = \{(x, z) :$ for some y, $R_1(x, y)$ and $R_2(y, z)\}$. Clearly, if R_1

is *EQ* then $R_1 \circ R_2 = R_2$, and if $R_2 = EQ$ then $R_1 \circ R_2 = R_1$. We use the standard notation for (possibly) disjunctive relations. The other possibilities are presented in the (composition) table of Figure 2(Top), where:

$\circ$	$lohp_\beta$	nhl_β	phl_β	$rohp_\beta$
$lohp_\alpha$	ct_0	ct_1	ct_2	?
nhl_α	ct_3	ct_4	ct_5	ct_6
phl_α	ct_1	ct_7	ct_8	ct_9
$rohp_\alpha$	?	ct_a	ct_b	ct_c

$\langle^{\imath}\alpha,\beta\rangle^{\jmath}$ s.t.	Translation of $\langle^{\imath}\alpha,\beta\rangle^{\jmath}(x,y)$
$\alpha \in [0,\pi), \beta \in [0,\pi)$	$(\langle lhp_\alpha\rangle^{\imath} \cap \langle rhp_\beta\rangle^{\jmath})(x,y)$
$\alpha \in [0,\pi), \beta \in [\pi,2\pi)$	$(\langle lhp_\alpha\rangle^{\imath} \cap \langle lhp_{\beta-\pi}\rangle^{\jmath})(x,y)$
$\alpha \in [\pi,2\pi), \beta \in [\pi,2\pi)$	$(\langle rhp_{\alpha-\pi}\rangle^{\imath} \cap \langle lhp_{\beta-\pi}\rangle^{\jmath})(x,y)$
$\alpha \in [\pi 2,\pi), \beta \in [0,\pi)$	$(\langle rhp_{\alpha-\pi}\rangle^{\imath} \cap \langle rhp_\beta\rangle^{\jmath})(x,y)$

Figure 2. (Top) Composition $R \circ S$, with R atom of $\mathcal{CAL}_\alpha$ and S atom of $\mathcal{CAL}_\beta$. (Bottom) Translation of basic relation $\langle^{\imath}\alpha,\beta\rangle^{\jmath}$ into $R \cap S$, with R (possibly disjunctive) $\mathcal{CAL}_\alpha$ relation, and S (possibly disjunctive) $\mathcal{CAL}_\beta$ relation.

- ct_0 is $lohp_\alpha$ if $\alpha = \beta$, ? otherwise;
- ct_1 is $lohp_\beta$ if $\alpha \geq \beta$, ? otherwise;
- ct_2 is $lohp_\alpha$ if $\alpha \leq \beta$, ? otherwise;
- ct_3 is $lohp_\beta$ if $\alpha \leq \beta$, ? othrwise;
- ct_4 is $lohp_\alpha \cap rohp_\beta$ if $\alpha > \beta$, nhl_α if $\alpha = \beta$, $rohp_\alpha \cap lohp_\beta$ otherwise;
- ct_5 is $lohp_\alpha \cap lohp_\beta$ if $\alpha < \beta$, $?_\alpha$ if $\alpha = \beta$, $rohp_\alpha \cap rohp_\beta$ otherwise;
- ct_6 is $rohp_\beta$ if $\alpha \geq \beta$, ? otherwise;
- ct_7 is $lohp_\alpha \cap lohp_\beta$ if $\alpha > \beta$, $?_\alpha$ if $\alpha = \beta$, $rohp_\alpha \cap rohp_\beta$ otherwise;
- ct_8 is $lohp_\alpha \cap rohp_\beta$ is $\alpha < \beta$, phl_α if $\alpha = \beta$, $rohp_\alpha \cap lohp_\beta$ otherwise;
- ct_9 is $rohp_\beta$ if $\alpha < \beta$, phl_α is $\alpha = \beta$, ? othrwise;
- ct_a is $rohp_\alpha$ if $\alpha \leq \beta$, ? otherwise;
- ct_b is $rohp_\alpha$ if $\alpha \geq \beta$, ? otherwise;
- ct_c is $rohp_\alpha$ if $\alpha = \beta$, ? otherwise;
- $? = \{(p,q) :\ p$ and q planar points$\}$ (i.e., ? is the universal binary relation on 2D points);
- $?_\alpha = \{(x,y) \in ?\ :\ x \in l(y,\alpha)\} = \{nhl_\alpha, EQ, phl_\alpha\}$ (i.e., the set of pairs (x,y) of 2D points s.t. $x \in l(y,\alpha)$).

It follows from the above that, given $\alpha,\beta \in [0,\pi)$, the composition $R_1 \circ R_2$ of $R_1 \in ATOMS(\alpha)$ and $R_2 \in ATOMS(\beta)$ is a convex relation.

It is now easy to derive the composition, $R \circ S$, of two *SCSP* basic constraints $R = \langle^{\imath_1}\alpha,\beta\rangle^{\jmath_1}$ and $S = \langle^{\imath_2}\gamma,\delta\rangle^{\jmath_2}$. It is sufficient to know how to translate an *SCSP* basic relation $R = \langle^{\imath}\alpha,\beta\rangle^{\jmath}$ into a conjunction $R_1 \cap R_2$, where R_1 is a $\mathcal{CAL}_\alpha$ or a $\mathcal{CAL}_{\pi-\alpha}$ convex relation, and R_2 a $\mathcal{CAL}_\beta$ or a $\mathcal{CAL}_{\pi-\beta}$ convex relation: this is done in the table of Figure 2(Bottom), where the following notation is used. Given $\alpha \in [0,\pi)$, we denote by $lchp_\alpha$ (resp. $rchp_\alpha$) the disjunctive relation $\{lohp_\alpha, nhl_\alpha, e, phl_\alpha\}$ (resp. $\{nhl_\alpha, e, phl_\alpha, rohp_\alpha\}$). The constraint $lchp_\alpha(x,y)$ (resp. $rchp_\alpha(x,y)$) means that x belongs to the Left (resp. Right) Close Half Plane bounded by $l(y,\alpha)$. Given $\alpha \in [0,\pi)$ and $\imath \in \{0,1\}$, the notation $\langle lhp_\alpha\rangle^{\imath}$ (resp. $\langle rhp_\alpha\rangle^{\imath}$) stands for $lohp_\alpha$ (resp. $rohp_\alpha$) if $\imath = 0$, and for $lchp_\alpha$ (resp. $rchp_\alpha$) if $\imath = 1$. It is important to keep in mind, when reading the table of Figure 2(Bottom), that $\alpha \in [\pi,2\pi)$ implies $(\alpha - \pi) \in [0,\pi)$.

The composition $R \circ S$ of basic constraints $R = \langle^{\imath_1}\alpha,\beta\rangle^{\jmath_1}$ and $S = \langle^{\imath_2}\gamma,\delta\rangle^{\jmath_2}$ can thus be written as $R \circ S = f(\alpha) \circ f(\gamma) \cap f(\alpha) \circ f(\delta) \cap f(\beta) \circ f(\gamma) \cap f(\beta) \circ f(\delta)$, where $f(x)$, for all $x \in \{\alpha,\beta,\gamma,\delta\}$, is a $\mathcal{CAL}_x$ atom if $x \in [0,\pi)$, and a $\mathcal{CAL}_{\pi-x}$ atom if $x \in [\pi,2\pi)$. Given that, for all $\alpha,\beta \in [0,\pi)$, the composition $R_1 \circ R_2$ of $R_1 \in ATOMS(\alpha)$ and $R_2 \in ATOMS(\beta)$ is a convex relation, we infer that the composition of two *SCSP* basic constraint is an *SCSP* convex constraint.

3.3 The intersection of two *SCSP* basic constraints

Clearly, $e \cap e = e$; $e \cap \langle^{\imath}\alpha,\beta\rangle^{\jmath} = e$ if $\imath = \jmath = 1$; and $e \cap \langle^{\imath}\alpha,\beta\rangle^{\jmath} = \emptyset$ if $\imath = 0$ or $\jmath = 0$.

Given a basic relation $R = \langle^{\imath}\alpha,\beta\rangle^{\jmath}$ and $\gamma \in [0,2\pi)$, γ is anticlockwisely inside R (notation $acwi(\gamma,R)$) *iff* (1) $\gamma = \alpha$ and $\imath = 1$; (2) $\gamma = \beta$ and $\jmath = 1$; or (3) $\gamma \neq \alpha$ and $\gamma \neq \beta$ and $\beta \ominus \alpha = (\gamma \ominus \alpha) + (\beta \ominus \gamma)$.

It is now easy to derive the intersection, $R \cap S$, of two *SCSP* basic constraints $R = \langle^{\imath_1}\alpha,\beta\rangle^{\jmath_1}$ and $S = \langle^{\imath_2}\gamma,\delta\rangle^{\jmath_2}$. If neither of $acwi(\alpha,S)$, $acwi(\beta,S)$, $acwi(\gamma,R)$ and $acwi(\delta,R)$ holds, then $R \cap S = \emptyset$. Otherwise, the intersection is nonempty: $R \cap S = \langle^{\imath}\phi,\theta\rangle^{\jmath}$. If $acwi(\alpha,S)$ then $\phi = \alpha$ and $\imath = \imath_1$, otherwise $\phi = \gamma$ and $\imath = \imath_2$. If $acwi(\beta,S)$ then $\theta = \beta$ and $\jmath = \jmath_1$, otherwise $\theta = \delta$ and $\jmath = \jmath_2$. Clearly, if $R \cap S \neq \emptyset$ then it is a basic constraint.

The converse of an *SCSP* basic constraint is an *SCSP* basic constraint. The composition of two *SCSP* basic constraints is either a basic constraint or the universal constraint. Finally, the intersection of two *SCSP* basic constraints is an *SCSP* basic constraint. Now, the only *SCSP* constraint that may (implicitly) appear in a *BSP* is, as already alluded to, the universal relation ?. Furthermore, the converse of ? is ?, $? \cap ?\ = ?$, $? \circ ?\ = ?$, and, for all basic relations R, $R \cap ? = ? \cap R = R$ and $R \circ ? = ? \circ R = ?$. This leads to the following theorem.

Theorem 1 *The class of* BSPs *is closed under path consistency: applying path consistency to a* BSP *either detects inconsistency of the latter, or leads to a (path consistent)* BSP.

It remains, however, to be proven that path consistency terminates when applied to a *BSP*. Furthermore, if path consistency is to be used as the filtering method during the search for a path consistent *BSP* refinement of a general *SCSP*, then it should also be proven that path consistency terminates when applied to a general *SCSP* -it may be worth noting here that path consistency applied to a general TCSP [6] may lead to what is known as the fragmentation problem [16]. We do this through the explanation of what we refer to as a "qualitative behaviour" of path consistency when applied to a general *SCSP*.

3.4 Qualitative behaviour of path consistency

Let P be a general *SCSP* and $HOLES(P)$ the set of all $\gamma \in [0,2\pi)$ such that there exists a constraint $[S_1 \vee \cdots \vee S_n](x,y)$ of P with, for some $i \in \{1,\ldots,n\}$, S_i of the form $\langle^{\imath}\alpha,\beta\rangle^{\jmath}$, and such that $\gamma \in \{\alpha,\beta\}$. We also denote by $HOLES^+(P)$ the set $HOLES(P) \cup \{\alpha \in [0,\pi) :\ (\alpha+\pi) \in HOLES(P)\} \cup \{\alpha \in [\pi,2\pi) :\ (\alpha-\pi) \in HOLES(P)\}$. Given a set A, we denote by $|A|$ the cardinality of A. Clearly $|HOLES^+(P)| \leq 2 \times |HOLES(P)|$. The qualitative behaviour comes from properties of the operations of converse, intersection and composition when applied to *SCSP* basic constraints. The intersection $R \cap S$ of two *SCSP* basic constraints $R = \langle^{\imath_1}\alpha,\beta\rangle^{\jmath_1}$ and $S = \langle^{\imath_2}\gamma,\delta\rangle^{\jmath_2}$ is of the form $\langle^{\imath}\phi,\theta\rangle^{\jmath}$, with both ϕ and θ in $\{\alpha,\beta,\gamma,\delta\}$. The converse of an *SCSP* basic constraint $\langle^{\imath}\alpha,\beta\rangle^{\jmath}$ is

	$\alpha = 0$	$0 < \alpha < \frac{\pi}{2}$	$\alpha = \frac{\pi}{2}$	$\frac{\pi}{2} < \alpha < \pi$
lohp$_\alpha(X,Y)$	$y_X > y_Y$	$y_X - y_Y > tg\alpha.(x_X - x_Y)$	$y_X < y_Y$	$y_X - y_Y > tg(\pi - \alpha).(x_X - x_Y)$
lchp$_\alpha(X,Y)$	$y_X \geq y_Y$	$y_X - y_Y \geq tg\alpha.(x_X - x_Y)$	$y_X \leq y_Y$	$y_X - y_Y \geq tg(\pi - \alpha).(x_X - x_Y)$
rohp$_\alpha(X,Y)$	$y_X < y_Y$	$y_X - y_Y < tg\alpha.(x_X - x_Y)$	$y_X > y_Y$	$y_X - y_Y < tg(\pi - \alpha).(x_X - x_Y)$
rchp$_\alpha(X,Y)$	$y_X \leq y_Y$	$y_X - y_Y \leq tg\alpha.(x_X - x_Y)$	$y_X \geq y_Y$	$y_X - y_Y \leq tg(\pi - \alpha).(x_X - x_Y)$

Figure 3. Translation of an *SCSP* basic constraint into a conjunction of linear inequalities.

$\langle^\imath \alpha \oplus \pi, \beta \oplus \pi\rangle^\jmath$: but such an operation will not create new "holes", since if a basic constraint of the form $\langle^\imath \alpha, \beta\rangle^\jmath(x,y)$ appears in P then we would have both α and β in *HOLES*(P), and both $\alpha \oplus \pi$ and $\beta \oplus \pi$ in *HOLES*$^+(P)$.

Path consistency using, as usual, a queue *QUEUE* where to put edges of P whose label has been updated, would thus, for each edge (i.e., pair of variables) (x,y), successfully update the label at most *HOLES*$^+(P)$ times. The number of edges is bounded by n^2, n being the number of variables of P. Furthermore, when an edge is taken from $Queue$ for propagation, $O(n)$ operations of converse, intersection and composition are performed. This leads to the following theorem stating termination, and providing a worst-case computational complexity, of path consistency applied to a general *SCSP*.

Theorem 2 *Applying path consistency to a general* SCSP P *with* n *variables terminates in* $O(|\mathrm{HOLES}(P)| \times n^3)$.

3.5 Translating an *SCSP* basic constraint into a conjunction of linear inequalities

We now provide a translation of an *SCSP* basic constraint into (a conjunction of) linear inequalities. We will then be able to translate any *BSP* into a conjunction of linear inequalities, and solve it with the well-known Simplex algorithm (see, e.g., [3]). This will give a complete solution search algorithm for general *SCSPs*, using path consistency at the internal nodes of the search space, as a filtering procedure, and the Simplex at the level of the leaves, as a completeness-guaranteeing procedure (the *SCSP* at the level of a leaf is a path-consistent *BSP*, but since we know nothing about completeness of path-consistency for *BSPs*, we need to translate into linear inequalities and solve with the Simplex).

Given a point X of the plane, we denote by (x_X, y_X) its coordinates. The translation of $e(X,Y)$ is obvious: $x_X - x_Y \leq 0 \wedge x_Y - x_X \leq 0 \wedge y_X - y_Y \leq 0 \wedge y_Y - y_X \leq 0$. For the translation of a general basic constraint $\langle^\imath \alpha, \beta\rangle^\jmath(X,Y)$, the results reported in the table of Figure 2(Bottom) imply that all we need is to show how to represent with a linear inequality each of the following relations on points X and Y, where $\alpha \in [0,\pi)$: *lohp*$_\alpha(X,Y)$; *lchp*$_\alpha(X,Y)$; *rohp*$_\alpha(X,Y)$; and *rchp*$_\alpha(X,Y)$. We split the study into four cases: $\alpha = 0$, $0 < \alpha < \frac{\pi}{2}$, $\alpha = \frac{\pi}{2}$, $\frac{\pi}{2} < \alpha < \pi$. The result is given by Figure 3, where, given an angle α, $tg\alpha$ denotes the tangent of α. We remind the reader that in a system of linear inequalities, there is a way of turning a strict inequality into a large one [3].

4 Summary

We have provided a *TCSP*-like decidable constraint language for reasoning about relative position of points of the 2-dimensional space. The language, *SCSPs* (Spatial Constraint Satisfaction Problems), subsumes two existing qualitative calculi of relations of cardinal directions [8], and is particularly suited for applications of large-scale high-level vision, such as, e.g., satellite-like surveillance of a geographic area. We have provided all the required tools for the implementation of the presented work; in particular, the algebraic operations of converse, intersection and composition, which are needed by path consistency. An adaptation of a solution search algorithm, such as, e.g., the one in [11] (see also [6]), which would use path consistency as the filtering procedure during the search, can be used to search for a path consistent *BSP* refinement of an input *SCSP*. But, because we know nothing about completeness of path consistency for *BSPs*, even when a path consistent *BSP* refinement exists, this does not say anything about consistency of the original *SCSP*. To make the search complete for *SCSPs*, we have proposed to augment it with the Simplex algorithm, by translating, whenever a leaf of the search space is successfully reached, the corresponding path consistent *BSP* into a conjunction of linear inequalities, which can be solved with the well-known Simplex algorithm [3].

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THE NOTIFICATION LETTER
(as received on 3 May 2004)

Dear Amar Isli:

We regret to inform you that your submission

C0689 A TCSP-like decidable constraint language generalising existing cardinal direction relations Amar Isli

cannot be accepted for inclusion in the ECAI 2004's programme. Due to the large number of submitted papers, we are aware that also otherwise worthwhile papers had to be excluded. You may then consider submitting your contribution to one of the ECAI's workshops, which are still open for submission.

In this letter you will find enclosed the referees' comments on your paper.

We would very much appreciate your participation in the meeting and especially in the discussions.

Please have a look at the ECAI 2004 website for registration details and up-to-date information on workshops and tutorials:

http://www.dsic.upv.es/ecai2004/

The schedule of the conference sessions will be available in May 2004.

I thanks you again for submitting to ECAI 2004 and look forward to meeting you in Valencia.

Best regards

Programme Committee Chair

REVIEW ONE

—– ECAI 2004 REVIEW SHEET FOR AUTHORS —–

PAPER NR: C0689

TITLE: A TCSP-like decidable constraint language generalising existing cardinal direction relations

1) SUMMARY (please provide brief answers)

- What is/are the main contribution(s) of the paper?

A new spatial reasoning calculus is proposed, combining the effectiveness of two (one qualitative, one quantitative) calculi. It is shown that path consistency is terminating on this calculus and this motivates a two stage solution technique where the simplex algorithm solves leaves of a path consistent instance.

2) TYPE OF THE PAPER

The paper reports on:

[ ] Preliminary research

[X] Mature research, but work still in progress

[ ] Completed research

The emphasis of the paper is on:

[X] Applications

[ ] Methodology

3) GENERAL RATINGS

Please rate the 6 following criteria by, each time, using only one of the five following words: BAD, WEAK, FAIR, GOOD, EXCELLENT

3a) Relevance to ECAI: EXCELLENT

3b) Originality: EXCELLENT

3c) Significance, Usefulness: FAIR

3d) Technical soundness: GOOD

3e) References: EXCELLENT

3f) Presentation: FAIR

4) QUALITY OF RESEARCH

4a) Is the research technically sound?

[X] Yes [ ] Somewhat [ ] No

4b) Are technical limitations/difficulties adequately discussed?

[X] Yes [ ] Somewhat [ ] No

4c) Is the approach adequately evaluated?

[ ] Yes [X] Somewhat [ ] No

FOR PAPERS FOCUSING ON APPLICATIONS:

4d) Is the application domain adequately described?

[ ] Yes [X] Somewhat [ ] No

4e) Is the choice of a particular methodology discussed?

[X] Yes [ ] Somewhat [ ] No

FOR PAPERS DESCRIBING A METHODOLOGY:

4f) Is the methodology adequately described?

[ ] Yes [ ] Somewhat [ ] No

4g) Is the application range of the methodology adequately described, e.g. through clear examples of its usage?

[ ] Yes [ ] Somewhat [ ] No

Comments:

This paper suggests an algorithm for solving a class of QSRs. In this case it would be useful to see some indication of problems solved used this technique that were hard to solve before.

5) PRESENTATION

5a) Are the title and abstract appropriate?

[ ] Yes [X] Somewhat [ ] No

5b) Is the paper well-organized? [ ] Yes [X] Somewhat [ ] No

5c) Is the paper easy to read and understand?

[ ] Yes [X] Somewhat [ ] No

5d) Are figures/tables/illustrations sufficient?

[X] Yes [ ] Somewhat [ ] No

5e) The English is [X] very good [ ] acceptable [ ] dreadful

5f) Is the paper free of typographical/grammatical errors?

[X] Yes [ ] Somewhat [ ] No

5g) Is the references section complete?

[X] Yes [ ] Somewhat [ ] No

Comments:

This paper is hard to read with much detailed technical content. The introduction which is for a KR paper is perhaps too long. More clarity could have been obtained with more effort in the technical sections.

6) TECHNICAL ASPECTS TO BE DISCUSSED (detailed comments)

- Suggested / required modifications:

A running example of an SCSP constraint network is necessary in this paper to illustrate the ideas. Partricularly the decomposition into lohp, rohp etc., The results of the path-consistency algorithm could then be demonstrated. It is hard to evaluate how effective path-consistency is at pruning in such networks.

- Other comments:

This paper needs to be clearer in exposition and technical details.

REVIEW TWO

—– ECAI 2004 REVIEW SHEET FOR AUTHORS —–

PAPER NR: C0689

TITLE: A TCSP-like decidable constraint language generalizing existing cardinal direction relations

1) SUMMARY (please provide brief answers)

- What is/are the main contribution(s) of the paper?

The paper describes a formalism for reasoning about directions in a 2D space with a fixed frame of reference. Basically it uses angular sectors analogous to intervals in the 1D case (the relevant calculus in one dimension being the formalism of Temporal constraint networks of Dechter, Meiri and Pearl.)

2) TYPE OF THE PAPER

The paper reports on:

[ ] Preliminary research

[X] Mature research, but work still in progress

[ ] Completed research

The emphasis of the paper is on:

[ ] Applications

[X] Methodology

3) GENERAL RATINGS

Please rate the 6 following criteria by, each time, using only one of the five following words: BAD, WEAK, FAIR, GOOD, EXCELLENT

3a) Relevance to ECAI: GOOD

3b) Originality: FAIR

3c) Significance, Usefulness: WEAK

3d) Technical soundness: WEAK

3e) References: GOOD

3f) Presentation: FAIR

4) QUALITY OF RESEARCH

4a) Is the research technically sound?

[ ] Yes [X] Somewhat [ ] No

4b) Are technical limitations/difficulties adequately discussed?

[ ] Yes [X] Somewhat [ ] No

4c) Is the approach adequately evaluated?

[ ] Yes [X] Somewhat [ ] No

FOR PAPERS FOCUSING ON APPLICATIONS:

4d) Is the application domain adequately described?

[ ] Yes [ ] Somewhat [ ] No

4e) Is the choice of a particular methodology discussed?

[ ] Yes [ ] Somewhat [ ] No

FOR PAPERS DESCRIBING A METHODOLOGY:

4f) Is the methodology adequately described?

[ ] Yes [X] Somewhat [ ] No

4g) Is the application range of the methodology adequately described, e.g. through clear examples of its usage?

[ ] Yes [ ] Somewhat [X] No

Comments:

I think reference should be made to the existing qualitative versions of direction calculi represented by Mitra 2000. In particular, as far as composing basic relations is concerned, Mitras paper (about qualitative relations) contains a simple description which is still basically valid in the quantitative case considered here.

In this respect, I do not think that the paper gives an adequate description of the calculus. The description of composition using the five relation calculus is convoluted and could be replaced by a much simpler one.

About the composition of two basic constraints: Using the description of a sector-like constraint (alpha,beta) in terms of the intersection of two half-plane constraints is a nice idea (you should give more intuition about that). But the resulting tables of Fig. 2 are not very easy to use: you could explain the results in a simpler way.

About theorem 1: What you prove is that, starting from basic constraints o(or the universal constraint) and doing composition and intersection results in the same kinds of relations. But this does not mean that you can detect inconsistency by path-consistency only. This would be a much deeper result!

5) PRESENTATION

5a) Are the title and abstract appropriate?

[X] Yes [ ] Somewhat [ ] No

5b) Is the paper well-organized? [X] Yes [ ] Somewhat [ ] No

5c) Is the paper easy to read and understand?

[ ] Yes [ ] Somewhat [X] No

5d) Are figures/tables/illustrations sufficient?

[X] Yes [ ] Somewhat [ ] No

5e) The English is [ ] very good [X] acceptable [ ] dreadful

5f) Is the paper free of typographical/grammatical errors?

[ ] Yes [X] Somewhat [ ] No

5g) Is the references section complete?

[ ] Yes [ ] Somewhat [ ] No

Comments:

6) TECHNICAL ASPECTS TO BE DISCUSSED (detailed comments)

- Suggested / required modifications:

I would have liked to recommend the paper for acceptance, because I think that it does contain some good ideas. Unfortunately, in its present state, it is still rather weak, because it does not give a sound description of the calculus.

As a consequence, I recommend rejection of the paper. But I think this work is worth a good overhaul which would make it into a valuable research contribution.

- Other comments:

Some typos:

On page 1 Column 1, line -4: computational instead of computatinnal. line -3: A system will feel happier?

On page 2 Column 1, line 6: constraints Line 30: with?

Column 2, line -14: framework instead of framewrok

On page 3

In the caption of Fig. 1, close should be replaced by closed.

Column 2, line 11: half plane