Local Appropriate Scale in Morphological Scale-Space

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Abstract. This paper discusses the problem of selecting appropriate scales for region detection *prior* to feature localization. We argue that an approach in morphological opening-closing scale-space is better than one in Gaussian scale-space. The proposed operator is based on a new shape decomposition method called morphological band-pass filter that decomposes an image into structures of different size *and* different curvature polarity. Local appropriate scale is then defined as the scale that maximizes the response of the band-pass filter at each point. This operator gives constant scale values in a region of constant width, and its zero-crossings coincide with local maxima of the gradient magnitudes. Its usefulness is demonstrated by some examples.

1 Introduction

Since their introduction by Witkin [14] scale-space representations have become a universal approach to a wide variety of computer vision tasks. They are based on the observation that real world objects and their projections onto images exist as meaningful entities only over certain ranges of scale. By making scale a parameter, an image can be transformed into a family of gradually simplyfied versions of itself. The scale parameter controls the amount of smoothing, thus the greater it is the more fine scale information is suppressed.

The most common implementation of this idea is the *Gaussian scale-space* which is defined by a convolution of the image $f(\mathbf{x})$ with a Gaussian kernel where the scale parameter determines the width of the Gaussian. The properties of this scale-space have been studied intensively by several researchers, see e.g. Lindeberg [8]. Since it is a "pure scale-space", i.e. it does not require any prior knowledge about the image content and treets all scales equally, an important question arises: If no scale is special in any way, how do we know at which scale level the interesting information can be found?

A very interesting answer to this question was given by Lindeberg [7]. He proposed to measure *local appropriate scales* which optimize the trade-off between smoothing and feature visibility. These measurements are then used to appropriately tune subsequent operators. Lindeberg defines the appropriate scale as the scale that maximizes the response of certain nonlinear operators w.r.t. scale. For example, a measure for the sizes of blobs and ridges, i.e. local extrema of image brightness, is obtained by maximizing (w.r.t. scale) the magnitudes of scale-normalized Laplaceans of Gaussian [7] or second directional derivatives of Gaussians [2], [4]. These operators give good results near the centers of blobs. However, near edges they reflect the sharpness of the edges rather than the blob sizes as is illustrated by figure 1 (center). Consequently, one has to localize blobs and edges *before* the results of these operators can be interpreted correctly.

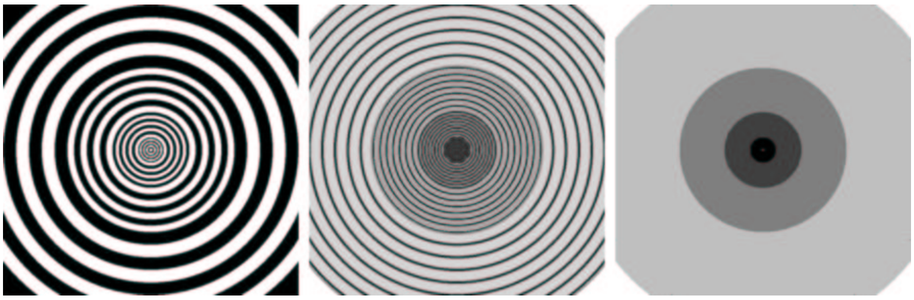

Fig. 1. Left: test image containing regions that have different constant widths, Center: appropriate scales measured by second directional derivatives are always small near edges, Right: magnitude of appropriate scales measured by our new method are constant within each region

In our opinion, appropriate scale measurements would be even more useful if they were available *prior* to feature detection. Hence we need an operator that works *uniformely* all over the image, regardless of what feature type a pixel belongs to. This could be achieved most naturally if appropriate scale were always associated with blob size, i.e. the edge response were supressed. In particlular all points in a region with constant width should have the same scale value - see the right image in figure 1. Two main problems must be solved:

1. A point may belong to regions at different scales simultaneously. These different regions must be identified, and the size of the most salient among them should determine the appropriate scale.
2. The width of a region must be defined and measured at every point without making unnecessary assumptions about possible region properties.

In this paper we propose to use *greyscale morphology* [12] to solve these problems. As opposed to convolution morphological operations are sensitive to geometrical shape. Morphological shape decomposition methods exploiting this fact have been developed by several researchers, e.g. [9], [11], [1], and [13]. Moreover, in [5] and [3] a solid theory of *morphological scale-space* is developed. This enables us to define local appropriate scales on the basis of morphological band-pass filters that will be defined as a generalisation of Wang et al. [13]. Due to space limitations all proofs have been omitted. Interested readers should refer to [6].

2 Morphological scale-space

2.1 Definition

Morphological scale-space has been developed into a coherent theory independently by Jackway [5] and van den Boomgaard and Smeulders [3]. Erosions and dilations or openings and closings are the basic operations needed to built the scale-space:

$$\begin{aligned} \text{Erosion:} \quad & (f \ominus g)(\mathbf{x}) = \inf_{\mathbf{x}' \in G} (f(\mathbf{x}+\mathbf{x}') - g(\mathbf{x}')) \\ \text{Dilation:} \quad & (f \oplus g)(\mathbf{x}) = \sup_{\mathbf{x}' \in G} (f(\mathbf{x}-\mathbf{x}') + g(\mathbf{x}')) \end{aligned} \tag{1}$$

$$\begin{aligned} \text{Opening:} \quad & (f \circ g)(\mathbf{x}) = ((f \ominus g) \oplus g)(\mathbf{x}) \\ \text{Closing:} \quad & (f \bullet g)(\mathbf{x}) = ((f \oplus g) \ominus g)(\mathbf{x}) \end{aligned} \tag{2}$$

The function $g(\mathbf{x})$ is called *structuring function*, and the region G its *support*.

Definition 1. A morphological opening-closing scale-space is defined [5] as

$$F(\mathbf{x}, s) = \begin{cases} (f \bullet g_s)(\mathbf{x}) & \text{if } s > 0 \\ f(\mathbf{x}) & \text{if } s = 0 \\ (f \circ g_{-s})(\mathbf{x}) & \text{if } s < 0 \end{cases} \tag{3}$$

The unification of opening and closing in one single scale-space with positive and negative scale values is possible because the two operations are *non-self-dual* - the former operates on local maxima of $f(\mathbf{x})$, while the latter operates on minima. The subscript at g_s indicates that we are using a family of structuring functions scaled by s ($g_s(\mathbf{x}) = g(\mathbf{x}/|s|)$). If the structuring function $g(\mathbf{x})$ is anti-convex the resulting scale-space satisfies a causality theorem, i.e. no new detail is introduced by the smoothing operations [5].

2.2 Morphological low-pass and high-pass filters

As we want to decompose images w.r.t. to region size we must choose the structuring function accordingly. To make as few assumptions as possible about the regions we model them as blobs, i.e. local extrema of $f(\mathbf{x})$ and their neighborhood[1]. Shape and size of a region can then be measured by anlysing the iso-contour lines each point belongs to. Formally, we get:

Definition 2. A point $\mathbf{x}_0$ constitutes a light (dark) blob or ridge of size s if there exists a closed disk D_s of radius s that contains $\mathbf{x}_0$ so that $f(\mathbf{x}) \geq f(\mathbf{x}_0)$ ($f(\mathbf{x}) \leq f(\mathbf{x}_0)$) for every point in the disk, and no such disk exists for any $s' > s$. All points in the disk are said to lie inside a blob of size s.

[1] Regions where the local extrema property holds only in certain directions will be called ridges.

Consequently, a point may lie inside blobs of different sizes. To identify the most salient of all possible blob sizes for a point will become the idea behind appropriate scale identification.

Definition 3. A low-pass filter with respect to blob size is characterized by the following properties (s is the limiting blob size of the filter):

1. The filter is isotropic.
2. Blobs smaller than $|s|$ are not present in the filtered image.
3. Blobs larger than $|s|$ are not be affected by the filtering.

A special case of property 3 is that a blob of infinite size, e.g. a single step edge, must not be changed for any finite $|s|$. This leads to the following proposition:

Proposition 4. *The only isotropic, anti-convex structuring functions that do not change a blob of infinite size under opening or closing are* the disks with radius s *(proof see [6]):*

$$g_s(\mathbf{x}) := d_s(\mathbf{x}) = \begin{cases} 0 & \text{if } |\mathbf{x}| \leq |s| \\ -\infty & \text{otherwise} \end{cases} \tag{4}$$

The relationship between blobs as defined above and opening-closing with disk structuring functions is established by the following proposition:

Proposition 5. *Morphological opening and closing with disk structuring functions* $d_s(\mathbf{x})$ *are perfect low-pass filters w.r.t. blob size. (proof see [6])*

Note also that only "flat" structuring functions (like disks) ensure invariance of morphological operations w.r.t. brightness scaling (i.e. $(\lambda f \circ g) = \lambda(f \circ g)$, see [10]). Therefore we will use disk structuring functions throughout this paper. Now we define a *high-pass with respect to feature size* by the relationship:

$$H(\mathbf{x}, s) = f(\mathbf{x}) - F(\mathbf{x}, s) \tag{5}$$

which gives rise to the following proposition:

Proposition 6. *An image morphologically high-pass filtered according to (5) does not contain blobs of size* s *and larger. (proof see [6])*

2.3 Morphological band-pass filters

In the next step we combine low- and high-pass filters to define a *band-pass filter with respect to blob size.*

Definition 7. A band-pass filter w.r.t. blob size has the following properties (s_l, s_u are lower and upper limiting sizes, $|s_l| < |s_u|$, $s_u s_l > 0$):

1. The filter should act isotropically.
2. Blobs smaller than $|s_l|$ are not present in the filtered image $B_{s_l}^{s_u}(\mathbf{x})$.
3. Blobs larger than $|s_u|$ are not present in the filtered image.

Generalizing an idea from Wang et al. [13] to our scale-space definition we get the following recursive algorithm that alternately high- and low-pass filters the image starting with high-pass filtering at the coarsest scales:

Proposition 8. *A family of perfect morphological band-pass filters with limiting blob sizes* $-\infty = s_{-n-1} < s_{-n} < \ldots < s_0 = 0 < \ldots < s_n < s_{n+1} = \infty$ *is obtained by the following formula* (s_n *must be larger than the image diagonal):*

$$\begin{aligned} \text{for } s_k \geq 0: \quad & H_{s_{n+1}}(\mathbf{x}) = f(\mathbf{x}) \\ & B_{s_k}^{s_{k+1}}(\mathbf{x}) = (H_{s_{k+1}} \bullet d_{s_k})(\mathbf{x}) \\ & H_{s_k}(\mathbf{x}) = H_{s_{k+1}}(\mathbf{x}) - B_{s_k}^{s_{k+1}}(\mathbf{x}) \\ \text{and for } s_k \leq 0: \quad & H_{s_{-n-1}}(\mathbf{x}) = f(\mathbf{x}) \\ & B_{s_k}^{s_{k-1}}(\mathbf{x}) = (H_{s_{k-1}} \circ d_{s_k})(\mathbf{x}) \\ & H_{s_k}(\mathbf{x}) = H_{s_{k-1}}(\mathbf{x}) - B_{s_k}^{s_{k-1}}(\mathbf{x}) \end{aligned} \tag{6}$$

where the resulting $B_{s_k}^{s_l}(\mathbf{x})$ *represent a morphological decomposition of the image into bands of different blob sizes and curvature polarities* (H_{s_k} *are intermediate high-pass filtered images). The original image can be exactly reconstructed from both the positive and the negative parts of the decomposition (proof see [6]):*

$$\sum_{k=0}^{n} B_{s_k}^{s_{k+1}}(\mathbf{x}) = \sum_{k=-n}^{0} B_{s_k}^{s_{k-1}}(\mathbf{x}) = f(\mathbf{x}) \tag{7}$$

Figure 2 shows a family of band-pass filtered images using (6). s is sampled in octaves. It is clearly visible how the image is decomposed into different structure sizes by the filter family.

3 Appropriate scale measurements in opening-closing scale-space

Similar to the proposal of Lindeberg [7], we identify the local appropriate scale as the scale that maximizes the response of a normalized band-pass filter with respect to scale:

Definition 9. The local appropriate scale of a blob at $\mathbf{x}$ is defined as:

$$s_A(\mathbf{x}) = \arg_{s_k} \left(\max_{\substack{s_k = s_{-n}, \ldots, s_{-1}, \\ s_1, \ldots, s_n}} \left| \frac{B_{s_{k\mp1}}^{s_k}(\mathbf{x})}{s_k - s_{k\mp1}} \right| \right) \tag{8}$$

where s_{k-1} applies if $s_k > 0$ and s_{k+1} if $s_k < 0$. The expression $B_{s_{k\mp1}}^{s_k}(\mathbf{x})/(s_k - s_{k\mp1})$ will be called normalized band-pass filter.

Figure 1 (right) shows the result of this operator on a test image. Note that the scale values within a region of constant width are constant. Figure 3 illustrates the application of the new appropriate scale operator to a natural image. Again the scale values correspond to the width of the region a pixel belongs

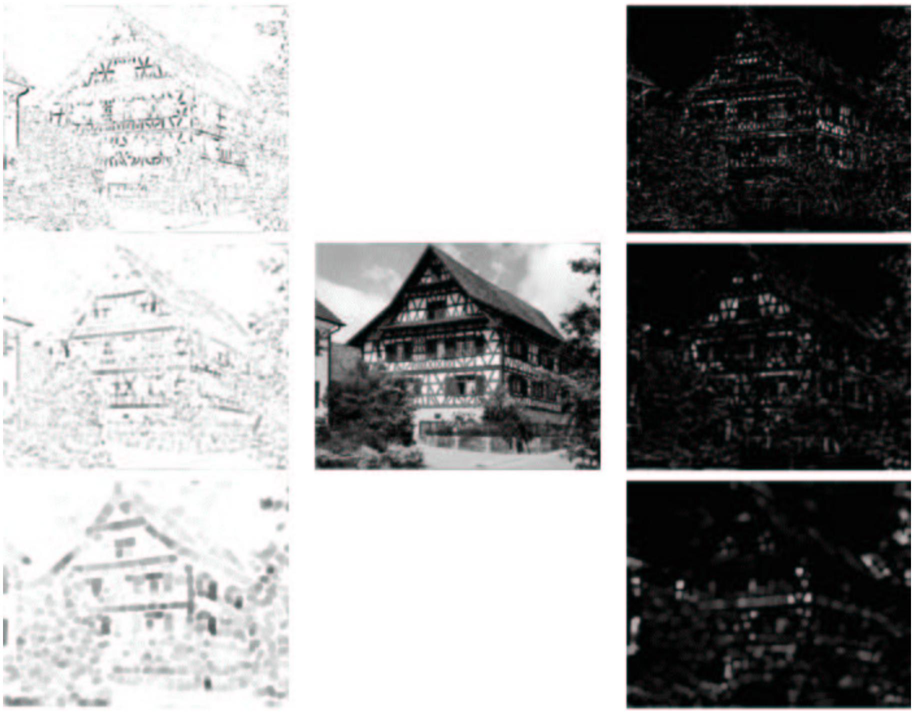

Fig. 2. Decomposition of an image (center) with respect to structure sizes. Left from top to bottom: $s = 2, 4, 8$ - dark blobs and ridges. Right: $s = -2, -4, -8$ - light blobs and ridges.

to. Scales are positive if this region is darker than its surroundings, negative otherwise.

Another fact is, however, somewhat surprising: The borders between areas of positive and negative scale ("zero-crossings" of the appropriate scale) correspond to image edges (local maxima of the image gradient). Although this behavior has been justified experimentally on a large number of images, we do not yet have a full theoretical explanation in 2D. An analysis of the scale operator in 1D indicates, however, that maxima of the normalized bandpass $B_{s_{k\mp1}}^{s_k}(x)/(s_k - s_{k\mp1})$ are indeed correlated with local maxima of the gradient $f'(x)$.

Consider the function in figure 4. The effect of morphological opening is best visualized by "fitting the structuring function under the original function". Opening with $d_s(x)$ therefore replaces the function between $f(x_1)$ and $f(x_2)$ with a straight line (where $x_2 - x_1 = 2s$). Likewise, opening with $d_{s'}(x)$ results in the straight line between $f(x'_1)$ and $f(x'_2)$ (with $x'_2 - x'_1 = 2s'$). Now normalized band-pass filtering between x_1 and x_2 yields

$$\left|\frac{B_s^{s'}(x)}{s' - s}\right| = \frac{f(x_1) - f(x'_1)}{s' - s} = \frac{f(x_2) - f(x'_2)}{s' - s}$$

Fig. 3. Appropriate scale measurements. For better visibility positive and negative scales are decomposed into two images (top: positive scales, bottom: magnitude of negative scales).

If we expand the r.h.s. into a Taylor series we arrive at

$$\lim_{s' \to s} \left| \frac{B_s^{s'}(x)}{s' - s} \right| = 2 \left(\frac{1}{|f'(x_1)|} + \frac{1}{|f'(x_2)|} \right)^{-1}$$

Hence the result of the morphological band-pass filter is proportional to the harmonic mean of the gradients at the points where the structuring function touches the original function. The gradients are maximized at edge points, thus the appropriate scale is obtained when the structuring function just fits between two edge points.

The following proposition establishes that the appropriate scale is invariant under rotation, translation, and brightness scaling while it scales accordingly when the spatial coordinates are uniformely scaled. This result is a major prerequisite for practical applications of approproate scales (like tuning of scale dependent operators towards suitable scales).

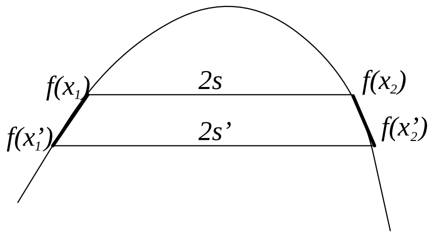

Fig. 4. Analysis of the appropriate scale operator in 1D (see text).

Proposition 10. *Let $\tilde{f}(\mathbf{x})$ be the image after similarity transformation and brightness rescaling, i.e. $\tilde{f}(\mu\mathbf{R}\mathbf{x}+\mathbf{x}_0) = \lambda f(\mathbf{x})$ where $\mathbf{R}$ denotes an arbitrary 2D rotation matrix, $\mathbf{x}_0$ an arbitrary translation, and $\lambda, \mu > 0$ are scalars. Then the appropriate scale $\tilde{s}_A$ of the transformed image $\tilde{f}(\mathbf{x})$ is given by (proof see [6]):*

$$\tilde{s}_A(\mu\mathbf{R}\mathbf{x} + \mathbf{x}_0) = \mu s_A(\mathbf{x}) \qquad (9)$$

4 Applications

4.1 Appropriate scale ridges

In this section we demonstrate that our new appropriate scale operator can be combined with Gaussian scale-space to select appropriate scales for ridge detection. We measure the scale-dependent ridge strength in Gaussian scale-space by the eigenvalues of the Hessian matrix ($\Phi_{xx}(\mathbf{x}, s)$ etc. denote second derivatives at scale s):

$$\gamma_{1,2}(\mathbf{x}, s) = \frac{1}{2}\left(\Phi_{xx}(\mathbf{x}, s) + \Phi_{yy}(\mathbf{x}, s) \pm \sqrt{(\Phi_{xx}(\mathbf{x}, s) - \Phi_{yy}(\mathbf{x}, s))^2 + 4\Phi_{xy}(\mathbf{x}, s)^2}\right) \qquad (10)$$

Now we use the appropriate scale $s_A(\mathbf{x})$ to select the correct scale, where the larger eigenvalue is taken for positive scales and the smaller for negative scales:

$$r(\mathbf{x}) = \begin{cases} \gamma_1(\mathbf{x}, s_A(\mathbf{x})) & (s_A(\mathbf{x}) > 0) \\ -\gamma_2(\mathbf{x}, -s_A(\mathbf{x})) & (s_A(\mathbf{x}) < 0) \end{cases} \qquad (11)$$

Since the appropriate scale depends only on the width of the ridge and does not change near edges, the ridge strength operator $r(\mathbf{x})$ shows very good step edge supression as is illustrated by figure 5.

4.2 Parameter-free binarization

The fact that the zero-crossings of the appropriate scales often coincide with image edges suggests to use the sign of the scales as a simple means to binarize the image:

Fig. 5. Appropriate scale ridges. Center: ridge strength, Right: ridge location after non-maxima supression

$$b(\mathbf{x}) = \begin{cases} 1 & (s_A(\mathbf{x}) < 0) \\ 0 & (s_A(\mathbf{x}) > 0) \end{cases} \quad (12)$$

This technique has two important advantages over thresholding: The binarization is invariant under brightness rescaling (Proposition 10) and the positions of the zero-crossings are insensitive to large scale shading. Figure 6 illustrates this by comparing classical thresholding with the new binarization method.

5 Conclusions

In this paper we discussed the problem of selecting appropriate scales *uniformly* at each pixel, regardless of the feature type the pixel belongs to. This led to a new appropriate scale operator built upon a morphological band-pass filter which proved more suitable to define uniform region based appropriate scale measurements than approaches based on Gaussian scale-space. In particular it has the following very interesting properties:

- The scale value obtained at any point does reflect the width of the most salient region the point belongs to.
- It is invariant w.r.t. brightness rescaling of the image.
- If the image undergoes a similarity transform it is scaled by the same amount as the spatial coordinates.
- The positions of zero-crossings of the appropriate scale seem to be correlated with maxima of the gradient magnitudes of the image.

Further investigation is needed to theoretically establish the last property in 2D.

Two applications illustrate that the new operator can indeed be used to improve other operators in making them invariant under similarity transformations and brightness scaling or tuning them towards the best scale of operation. We expect very interesting results from further research in this direction.

Fig. 6. Left: conventional thresholding with hand-tuned threshold, Right: parameter-free binarization

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