Morphological Appropriate Scale Measurements for Region Segmentation

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Abstract

This paper presents a novel approach to selecting appropriate scales in morphological opening-closing scale-space. It is based on a morphological band-pass filter that decomposes an image into structures of different size *and* different curvature polarity ("light and dark blobs"). Appropriate scale is defined as the scale that maximizes the response of the bandpass. The resulting scale measurements allow to automatically select window sizes (scales) for segmentation operators. The application of this idea to region segmentation gives very satisfying results.

1 Introduction

When extracting low-level features from images, traditionally one has to adjust a number of parameters, most notably window and filter sizes. Often the adjustment of these parameters is done by hand thus adapting a certain algorithm to a particular class of images (or in some cases just one particular image). The elimination of these parameters (or their automatic adaption) has been the focus of much research in the past.

A new basis for the solution of this problem was laid by the observation that choosing window sizes is actually a problem of scale selection: Any image feature is only visible over a certain range of scale. By introducing a new dimension that represents scale we can transform the image into a family of gradually simplifyed versions of itself, the so called scale-space (Witkin 1983 [9]).

Depending on the particular algorithm used for smoothing different scale-spaces will result. Gaussian scale-space is the only linear and thus most common variant of this idea. It is defined by a convolution of the image $f(\vec{x})$ with a Gaussian kernel where the scale parameter determines the width of the Gaussian. A number of nonlinear scale-spaces have also been defined, and the morphological opening-closing scale-space will be used in this paper.

Based on the scale-space representation of the image Lindeberg [7] proposed an elegant solution to our original problem of selecting appropriate window sizes (scales) for feature detection. He defines a *local appropriate scale* as the scale that maximizes the salience of certain image properties w.r.t. scale. These scale measurements are then used to tune subsequent feature detectors towards their optimal window size. For example, a measure for the sizes of blobs and ridges, i.e. local extrema of image brightness, is obtained by maximizing (w.r.t. scale) the magnitudes of the scale normalized Laplacean of Gaussian (Lindeberg [7]) or the magnitudes of second directional derivatives of Gaussians (Burns et al. [2], Dana and Wildes [3]).

These functions give good results near the centers of blobs. However, near edges they reflect the sharpness of the edges rather than the sizes of the nearest blob as is illustrated by figure 1 (center). Consequently, one has to localize blobs resp. edges *before* the results of these operators can be interpreted correctly.

However, to adapt to appropriate scales completely automatically we need an operator that can be applied prior to feature detection and works uniformely all over the image. This could be achieved most naturally if appropriate scale

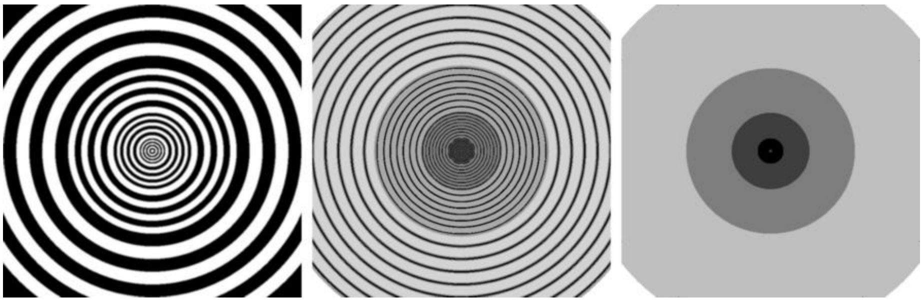

Figure 1: Left: test image containing regions that have different constant widths, Center: appropriate scales measured by eq. (3) are always small near edges, Right: magnitude of appropriate scales measured by the new method are constant within each region

were always associated with blob size, i.e. the edge response were supressed. In particlular all points in a region with constant width should have the same scale value - see the right image in figure 1. Two main problems must be solved:

1. A point may belong to regions at different scales simultaneously. These different regions must be identified, and the size of the most salient among them should determine the appropriate scale.
2. The width of a region must be defined and measured at every point without making unnecessary assumptions about possible region properties.

In a recent paper [6] we proposed to use *greyscale morphology* to solve these problems because morphological operations are more sensitive to geometrical shape than convolution based operators. Here we will summarize the theoretical results from this paper and cover the aplication aspects in more detail. In particular, an intergrated algorithm that uses the appropriate scale measurements for region segmentation will be described.

2 Morphological opening-closing scale-space

Morphological opening-closing scale-space has been developed into a coherent theory by Jackway [4]. The basic operations are defined as follows:

$$\begin{aligned} \text{Erosion:} \quad (f \ominus g)(\vec{x}) &= \inf_{\vec{x}' \in G} \left(f(\vec{x}+\vec{x}') - g(\vec{x}')\right) \\ \text{Dilation:} \quad (f \oplus g)(\vec{x}) &= \sup_{\vec{x}' \in G} \left(f(\vec{x}-\vec{x}') + g(\vec{x}')\right) \qquad (1) \\ \text{Opening:} \quad (f \circ g)(\vec{x}) &= \left((f \ominus g) \oplus g\right)(\vec{x}) \\ \text{Closing:} \quad (f \bullet g)(\vec{x}) &= \left((f \oplus g) \ominus g\right)(\vec{x}) \qquad (2) \end{aligned}$$

The function $g(\vec{x})$ is called *structuring function*, and the region G its *support*. We requires $g(\vec{x}) \leq 0$ and $g(\vec{0}) = 0$ so that a constant function $f(\vec{x}) = const$ will not be altered by the morphological operations. Opening/closing are dual operations, i.e.:

$$(f \circ g)(\vec{x}) = -\left((-f) \bullet \check{g}\right)(\vec{x}) \qquad (3)$$

and vice versa ($\check{g}(\vec{x}) = g(-\vec{x})$).

Definition 1 *A morphological opening-closing scale-space is defined [4] as*

$$F(\vec{x}, s) = \begin{cases} (f \bullet g_s)(\vec{x}) & \text{if } s > 0 \\ f(\vec{x}) & \text{if } s = 0 \\ (f \circ g_{-s})(\vec{x}) & \text{if } s < 0 \end{cases} \tag{4}$$

The unification of opening and closing in one single scale-space with positive and negative scale values is possible because the two operations are *non-self-dual* - the former operates on local maxima of $f(\vec{x})$, while the latter operates on minima. The subscript at g_s indicates that we are using a family of structuring functions scaled by s ($g_s(\vec{x}) = g(\vec{x}/|s|)$). If all those structuring functions are anti-convex the resulting scale-space satisfies a causality theorem, i.e. no new detail is introduced by the smoothing operations [4].

3 Morphological bandpass filters

The general definition if morphological scale-space does not yet specify which structuring functions should be chosen. In [6] we proved that an appropriate choice allows us to interpret the opening-closing scale-space as a lowpass filter w.r.t. region size. Here we will outline this only briefly.

Regions shall be represented by blobs (local maxima or minima of the greylavel) which are formally defined as:

Definition 2 *A point $\vec{x}_0$ constitutes a light blob or ridge of size s if there exists a closed disk D_s of radius s that contains $\vec{x}_0$ so that $f(\vec{x}) \geq f(\vec{x}_0)$ for every point in the disk, and no such disk exists for any $s' > s$. Conversely, for dark blobs or ridges we require $f(\vec{x}) \leq f(\vec{x}_0)$ everywhere in the disk. All points in the disk are said to lie inside a blob of size s.*

Consequently, a point may lie inside blobs of different sizes. To identify the most salient of all possible blob sizes for a point will become the idea behind appropriate scale identification.

Definition 3 *A low-pass filter with respect to blob size is characterized by the following properties (s is the limiting blob size of the filter):*

1. *The filter is isotropic.*
2. *Blobs smaller than $|s|$ are not present in the filtered image.*
3. *Blobs larger than $|s|$ are not be affected by the filtering.*

A special case of property 3 is that a blob of infinite size, e.g. a single step edge, must not be changed for any finite $|s|$. This leads to the following proposition:

Proposition 1 *The only isotropic, anti-convex structuring functions that do not change a blob of infinite size under opening or closing are* the disks with radius s *(proof see [6]):*

$$g_s(\vec{x}) := d_s(\vec{x}) = \begin{cases} 0 & \text{if } |\vec{x}| \leq |s| \\ -\infty & \text{otherwise} \end{cases} \tag{5}$$

The relationship between blobs as defined above and the opening-closing operations is established by the following proposition:

Proposition 2 *Morphological opening and closing with disk structuring functions d_s are perfect low-pass filters w.r.t. blob size. (proof see [6])*

Now we use this property to build a morphological bandpass filter similar to the well known Difference-of-Gaussian filter in Gaussian scale-space.

Definition 4 *A band-pass filter w.r.t. blob size has the following properties (s_l and s_u are lower and upper limiting sizes with $|s_l| < |s_u|$ and $s_u s_l > 0$):*

1. *The filter should act isotropically.*
2. *Structures smaller than $|s_l|$ are not present in the filtered image $B_{s_l}^{s_u}(\vec{x})$.*
3. *Structures larger than $|s_u|$ are not present in the filtered image.*

Generalizing an idea from Wang et al. [8] to our scale-space definition we get the following recursive algorithm that alternately high- and low-pass filters the image starting with high-pass filtering at the coarsest scales:

Proposition 3 *A family of perfect morphological band-pass filters with limiting blob sizes $-\infty = s_{-n-1} < s_{-n} < \ldots < s_0 = 0 < \ldots < s_n < s_{n+1} = \infty$ is obtained by the following formula (s_n must be larger than the image diagonal):*

$$\begin{array}{rrcl} \text{for } s_k \geq 0: & H_{s_{n+1}}(\vec{x}) & = & f(\vec{x}) \\ & B_{s_k}^{s_{k+1}}(\vec{x}) & = & (H_{s_{k+1}} \bullet d_{s_k})(\vec{x}) \\ & H_{s_k}(\vec{x}) & = & H_{s_{k+1}}(\vec{x}) - B_{s_k}^{s_{k+1}}(\vec{x}) \\ \text{and for } s_k \leq 0: & H_{s_{-n-1}}(\vec{x}) & = & f(\vec{x}) \\ & B_{s_k}^{s_{k-1}}(\vec{x}) & = & (H_{s_{k-1}} \circ d_{s_k})(\vec{x}) \\ & H_{s_k}(\vec{x}) & = & H_{s_{k-1}}(\vec{x}) - B_{s_k}^{s_{k-1}}(\vec{x}) \end{array} \tag{6}$$

where the resulting $B_{s_k}^{s_l}(\vec{x})$ represent a morphological decomposition of the image into bands of different blob sizes and curvature polarities (H_{s_k} are intermediate high-pass filtered images). The original image can be exactly reconstructed from both the positive and the negative parts of the decomposition (proof see [6]):

$$\sum_{k=0}^{n} B_{s_k}^{s_{k+1}}(\vec{x}) = \sum_{k=-n}^{0} B_{s_k}^{s_{k-1}}(\vec{x}) = f(\vec{x}) \tag{7}$$

4 Appropriate scale measurements in opening-closing scale-space

Similar to the proposal of Lindeberg [7], we identify the local appropriate scale as the scale that maximizes the response of a normalized band-pass filter with respect to scale:

Definition 5 *The local appropriate scale of the image at $\vec{x}$ is defined as:*

$$s_A(\vec{x}) = \arg_{s_k}\left(\max_{\substack{s_k = s_{-n},\ldots,s_{-1},\\ s_1,\ldots,s_n}} \left|\frac{B_{s_{k\mp1}}^{s_k}(\vec{x})}{s_k - s_{k\mp1}}\right|\right) \tag{8}$$

where s_{k-1} applies if $s_k > 0$ and s_{k+1} if $s_k < 0$. The expression $B_{s_{k\mp1}}^{s_k}(\vec{x})/(s_k - s_{k\mp1})$ will be called normalized band-pass filter.

Figure 1 (right) shows the result of this operator on a test image (brightness encodes the *magnitude* of the appropriate scale). Note that the scale values within a region of constant width are constant. Figure 2 illustrates the application of the new appropriate scale operator to a natural image. As in the test image, the scale values correspond to the width of the region a pixel belongs

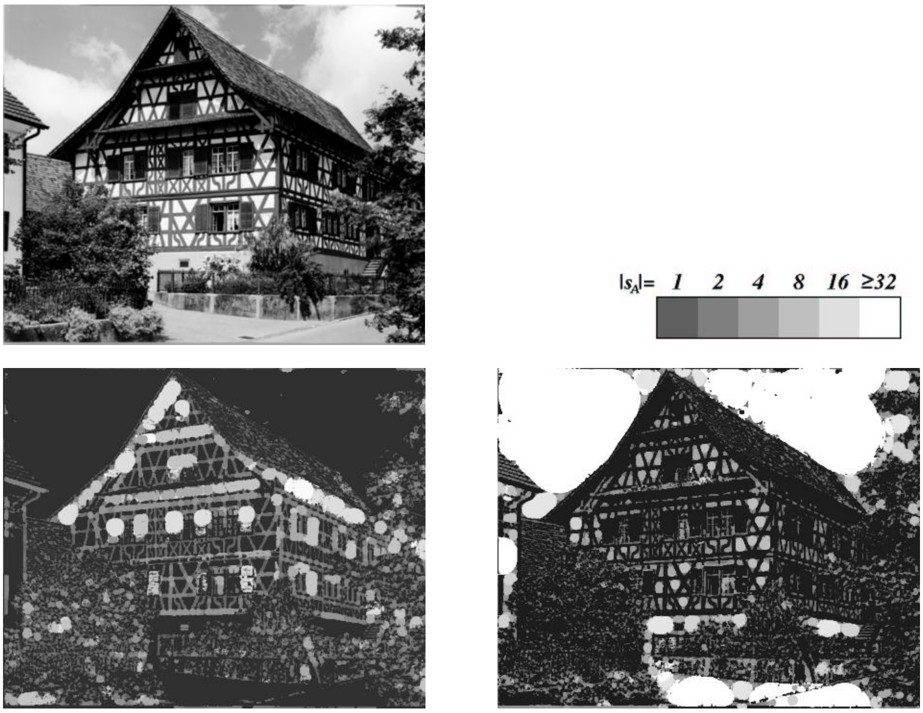

Figure 2: Appropriate scale measurements in an example image. For better visibility positive and negative scales are decomposed into two images (left: positive scales, right: magnitude of negative scales).

to. Scales are positive if this region is darker then its surroundings, negative otherwise. Another fact is, however, somewhat surprising: The borders between areas of positive and negative scale ("zero-crossings" of the appropriate scale) correspond to image edges (local maxima of the image gradient). Although this very desirable behavior (which can be used to obtain a parameter-free binarization) has been justified experimentally on a large number of images, we do not yet have a full theoretical explanation in 2D. An analysis of the scale operator in 1D indicates, however, that the maximum of the normalized bandpass $B^{s_k}_{s_{k\mp1}}(x)/(s_k - s_{k\mp1})$ is indeed correlated with local maxima of the gradient $f'(x)$ (see [6]).

5 Application to Region Segmentation

We now use the appropriate scale measurements to derive paremeter-free egde detection and ridge detection methods. These are then combined into a seeded region growing algorithm as described in [5].

The observation that zero crossings of the approriate scales coincide with local maxima of the gradients suggests that we apply a standard zero crossing detector to the scale image to get an *parameter free edge operator*. Figure 3 left shows the result of this operator on a subregion of figure 2. Regions can be determined by inverting the edge image. Due to missing edges these regions tend to be undersegmented.

A *parameter free ridge operator* is obtained by tunig a standard ridge detector in Gaussian scale-space towards the appropriate scales measured in morphological scale-space. We measure the ridge strength by the eigenvalues of the

Figure 3: Left: Edges obtained as zero crossings of the appropriate scale image (detail). Right: Nonmaxima suppression of appropriate scale ridges (black: maxima, white: minima) (detail)

Hessian matrix ($\Phi_{xx}(\vec{x}, s)$ etc. denote second derivatives at scale s in Gaussian scale-space):

$$\mathbf{H}(\vec{x}, s) = \begin{pmatrix} \Phi_{xx}(\vec{x}, s) & \Phi_{xy}(\vec{x}, s) \\ \Phi_{xy}(\vec{x}, s) & \Phi_{yy}(\vec{x}, s) \end{pmatrix}$$

The eigenvalues are functions of scale:

$$\gamma_{1,2}(\vec{x}, s) = \frac{1}{2}\left(\Phi_{xx}(\vec{x}, s) + \Phi_{yy}(\vec{x}, s) \pm \sqrt{(\Phi_{xx}(\vec{x}, s) - \Phi_{yy}(\vec{x}, s))^2 + 4\Phi_{xy}(\vec{x}, s)^2}\right) \tag{9}$$

Now we use the appropriate scale $s_A(\vec{x})$ to select the correct scale of the ridges, where the larger eigenvalue is used for positive scales and the smaller for negative scales:

$$r(\vec{x}) = \begin{cases} \gamma_1(\vec{x}, s_A(\vec{x})) & (s_A(\vec{x}) > 0) \\ \gamma_2(\vec{x}, -s_A(\vec{x})) & (s_A(\vec{x}) < 0) \end{cases} \tag{10}$$

Since the appropriate scale depends only on the width of the ridge and does not change near edges, this operator shows a very good step edge supression. The actual ridges are now found by a standard non-maxima suppression in the direction of largest curvature (figure 3 right).

One problem with this technique has not been resolved completely: The appropriate scales measured within one region change rapidly when the region's width changes rapidly. Due to the sampling of the scale coordinate this causes a jump in the scale measurements (see for example the sky in figure 2). This in turn causes a jump in the ridge strength measurements which may lead to incorrect maxima detection (typically several collinear ridges are found instead of one long ridge). Currently we are simply smoothing the measured scales to avoid this effect. However, this does not always work, so a better technique is still needed.

To overcome the shortcomings of these two operators they are combinded using a technique described in [5]. The segmentation is partitioned into two steps: seed selection and region growing. Seed selection results in a partial segmentation, where all regions of the final segmentation are already present, but only a few pixels ("seeds") are assigned to each region, while most pixels remain unlabeled. The actual shape of each region is then found using seeded region growing [1]: The similarity of pixels w.r.t to an adjacent region (seed) is measured using an appropriate fitness function (e.g. the local gradient). These

Figure 4: Left: Seeds used to start region growing. Right: Resulting segmentation.

similarity values are inserted into a global priority queue so that pixels with high similarity are aggregated first. This procedure ensures that regions always meet at the pixels that are most distinct from all their neighboring regions. If the initial seeds represented the desired regions correctly (in particular there was the right number of seeds) the estimated shapes are very accurate (see [5]).

The process of seed selection is critical for the success of the segmentation. Therefore [5] proposed a method to combine different sources of seeds according to their reliability, namely:

1. homogeneous regions. These regions can be found easily by thresholding the gradients of the image, where the threshold is determined by the noise distribution as measured in the gradient histogramm. This method produces oversegmentation (if a conservative threshold is chosen), so we use it only for large regions that are hard to find with the other methods due to the large windows that were required then.

2. regions detected by inverting an edge map. These regions tend to be undersegmented due to missed edge segments. The probability that a region contains missed edges is very low if the region is (almost) convex, so only those regions are used as seeds.

3. ridges detected by an ridge detection algorithm. These seeds tend to produce oversegmentation thus we use them only where none of the other seeds is available.

Using the edge and ridge detection methods outlined above the combined segmentation algorithm can be adjusted to varying scales in the image. The seeds used and the resulting segmentation are shown in figure 4. The accuracy of the segmentation can be seen in figure 5.

6 Conclusions

In this paper we discussed the problem of selecting appropriate window sizes, i.e. appropriate scales for low-level segmentation algorithms. The lead to the need of an operator that works prior to feature detection and uniformely all over the image.

Generalizing morphological decomposition methods we developed a morphological band-pass filter in opening-closing scale-space. It decomposes images into geometrical structures of different sizes *and* different curvature ("dark and

Figure 5: Accuracy of segmentation: edges overlaid over original image.

light blobs") and may be viewed as a morphological analogy to the well-known Laplacian of Gaussian operator.

Our new appropriate scale operator built upon this morphological band-pass filter fulfills the outlined reqirements and proved suitable to tune different feature detectors towards appropriate scales. Using this idea we defined parameter free edge and ridge detection operators. Combining these operators according to their strengths resulted in a very satisfying segmentation. Improvements are still possible by further improving the basic operators and by better understanding and measuring their specific strengths and weaknesses.

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