On Terminological Default Reasoning about Spatial Information: Extended Abstract

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Abstract

We extend the theory about terminological default reasoning using a logical base language that can be used to represent spatioterminological phenomena. Based on the description logic $\mathcal{ALCRP}(\mathcal{S}_2)$ the paper discusses how to implement an algorithm for computing extensions of a world description consisting of $\mathcal{ALCRP}(\mathcal{S}_2)$ assertions and a set of closed $\mathcal{ALCRP}(\mathcal{S}_2)$ Reiter-style default rules.

1 Introduction

This paper investigates a Reiter-based approach to terminological default reasoning about spatial information. Originally, a default rule has the form $\alpha : \beta_1, \beta_2, \ldots, \beta_n/\gamma$ where α, β_i and γ are FOPL formulae. α is called the *precondition* of the rule, the β_i terms are called *justifications,* and γ is the *consequent.* Intuitively the idea behind default reasoning is the following: starting with a world description A of what is known to be true, default rules can be applied such that they augment A by default rule conclusions γ to yield a *set of beliefs.* A default can be applied, i.e. its conclusion γ can be added to the set of current beliefs iff α is entailed by this set, each formula β_i is consistent with the current set of beliefs and γ is not already entailed. In our case the idea is that α, β_i and γ are special $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes which contain only *concept assertions* (that fulfill the $\mathcal{ALCRP}(\mathcal{D})$ restrictedness criteria) as well as *complex role assertions* [3].

Defaults may interact and depending on the set of default rules being applied, different "possible worlds" can be computed. These possible worlds are referred to as *extensions* (see below for a formal definition). Depending on the reasoning mode the *consequence problem* for terminological default theories is to decide whether a given assertional axiom is member of all extensions (skeptical mode) or of at least one extension (credulous mode).

Using description logic *concept terms* in default rules instead of first-order or propositional logic formulae has been extensively considered in [1]. A *terminological default theory* is a pair (A, D) where A is an ABox, and D is a finite set of *terminological* default rules whose preconditions, justifications and consequents are concept terms. Because concept terms correspond to unary predicates ranging over a free variable, these defaults are called *open* defaults. In contrast, *closed* defaults do not contain any free variables. Unlike Reiter's original proposal, the approach of [1] applies defaults only to those individuals that are explicitly mentioned in the world description (ABox). Default rules are never applied to implicit individuals introduced by $\exists$-restrictions. With this kind of semantics the consequence problem for (A, D) is decidable (see [1] for details). Closed default rules can be obtained by instantiating the free variable in the concept expressions with all explicitly mentioned ABox individuals (see [1] for a formal definition). Thus, for closed defaults, α, β_i and γ are *concept* assertions (ABox concept axioms). Once we have a closed default theory, a set of consequences of such a theory is referred to as an *extension* which is a set of deductively closed formulae defined by a fixed point construction. In the case of terminological default reasoning about spatial information it is also interesting to conclude spatial relations by default. Therefore, we extended the approach presented in [1] to deal with role assertions in default rules. Before discussing the computation of extensions of closed default theories in the case where α, β_i and γ are $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes consisting exclusively of $\mathcal{ALCRP}(\mathcal{S}_2)$ *concept assertions* and *complex role assertions*, we first consider an example of using defaults in the context of terminological reasoning about spatial information.

2 $\mathcal{ALCRP}(\mathcal{S}_2)$ Preliminaries

$\mathcal{ALCRP}(\mathcal{S}_2)$ is the description logic resulting from the instantiation of the description logic $\mathcal{ALCRP}(\mathcal{D})$ with the concrete domain $\mathcal{D} = \mathcal{S}_2$ (see [3, 4]). $\mathcal{ALCRP}(\mathcal{D})$ extends $\mathcal{ALC}(\mathcal{D})$ by a role-forming predicate-based operator, whose semantics is given by

$$\begin{array}{l}(\exists(u_1,\ldots,u_n)(v_1,\ldots,v_m).P)^{\mathcal{I}} := \\ \quad\{(a,b)\in\Delta_{\mathcal{I}}\times\Delta_{\mathcal{I}} \mid \exists x_1,\ldots,x_n,y_1,\ldots,y_m\in\Delta_{\mathcal{D}}: \\ \qquad (a,x_1)\in u_1^{\mathcal{I}},\ldots,(a,x_n)\in u_n^{\mathcal{I}}, \\ \qquad (b,y_1)\in v_1^{\mathcal{I}},\ldots,(b,x_m)\in v_m^{\mathcal{I}}, \\ \qquad (x_1,\ldots,x_n,y_1,\ldots,y_m)\in P^{\mathcal{D}}\}\end{array}$$

where P is a concrete-domain predicate name, u_i and v_j are feature chains, $\Delta_{\mathcal{D}}$ is the universe of the concrete domain, $\Delta_{\mathcal{I}}$ is the universe of the abstract domain. Informally, $(a,b) : \exists(u_1,\ldots,u_n)(v_1,\ldots,v_m).P$ holds iff there exists the appropriate concrete domain objects $x_1,\ldots,x_n,y_1,\ldots,y_m$ reachable via the feature chains $u_1,\ldots,u_n$ from a (resp. $v_1,\ldots,v_m$ from b) such that the predicate $P(x_1,\ldots,x_n,y_1,\ldots,y_m)$ holds.

In the case of $\mathcal{D}=\mathcal{S}_2$ we have $n,m=1$, and, as an ontological decision, we call $u_1=v_1=has\_area$ (a single feature is considered as a feature chain of length one).

Definition 1. *The concrete domain $\mathcal{S}_2$ is defined w.r.t. the topological space $\langle\mathbb{R}^2,2^{\mathbb{R}^2}\rangle$. The domain $\Delta_{\mathcal{S}_2}$ contains all non-empty, regular closed subsets of $\mathbb{R}^2$ which are called* regions *for short. The set of predicate names is defined as follows:*

- *A unary concrete_domain_top predicate* is-region *with* $\text{is-region}^{\mathcal{S}_2}=\Delta_{\mathcal{S}_2}$ *and its negation* is-no-region *with* $\text{is-no-region}^{\mathcal{S}_2}=\emptyset$.
- *The 8 basic predicates* dc*,* ec*,* po*,* tpp*,* ntpp*,* tppi*,* ntppi *and* eq *correspond to the RCC-8 relations. Due to space restrictions we would like to refer to [4] for a formal definition of the semantics.*
- *In order to name disjunctions of base relations, we need additional predicates. Unique names for these "disjunction predicates" are enforced by imposing the following canonical order on the basic predicate names:* dc*,* ec*,* po*,* tpp*,* ntpp*,* tppi*,* ntppi*,* eq*. For each sequence $p_1,\ldots,p_n$ of basic predicates in canonical order ($n\geq 2$), an additional predicate of arity 2 is defined. The predicate has the name* $\mathsf{p_1}\text{-}\cdots\text{-}\mathsf{p_n}$ *and we have $(r_1,r_2)\in \mathsf{p_1}\text{-}\cdots\text{-}\mathsf{p_n}^{\mathcal{S}_2}$ iff $(r_1,r_2)\in\mathsf{p_1}^{\mathcal{S}_2}$ or ... or $(r_1,r_2)\in\mathsf{p_n}^{\mathcal{S}_2}$. The predicate* dc-ec-po-tpp-ntpp-tppi-ntppi-eq *is also called* spatially-related.
- *A binary predicate* inconsistent-relation *with* $\text{inconsistent-relation}^{\mathcal{S}_2}=\emptyset$ *is the negation of* spatially-related.

Proposition 1. $\mathcal{S}_2$ *is admissible* (see [4]).

3 Spatioterminological Default Reasoning: An Example

Using $\mathcal{ALCRP}(\mathcal{S}_2)$'s role-forming predicate-based operator, we define a set of complex roles according to the mentioned RCC-8 $\mathcal{S}_2$ predicates:

$$\begin{array}{rl}inside \doteq & \exists(has\_area)(has\_area).\mathsf{tpp\text{-}ntpp} \\ contains \doteq & \exists(has\_area)(has\_area).\mathsf{tppi\text{-}ntppi} \\ overlaps \doteq & \exists(has\_area)(has\_area).\mathsf{po} \\ touches \doteq & \exists(has\_area)(has\_area).\mathsf{ec} \\ disjoint \doteq & \exists(has\_area)(has\_area).\mathsf{dc}\end{array}$$

The following definitions of concepts required to model domain objects representing different kinds of regions in a TBox satisfy the $\mathcal{ALCRP}(\mathcal{D})$ restrictedness criteria.

$$\begin{array}{rl}area \doteq & \exists has\_area.\mathsf{is\text{-}region} \\ natural\_region \doteq & \neg administrative\_region \\ country\_region \dot\sqsubseteq & administrative\_region \sqcap \\ & large\_scale \sqcap area \\ city\_region \dot\sqsubseteq & administrative\_region \sqcap \\ & \neg large\_scale \sqcap area \\ lake\_region \dot\sqsubseteq & natural\_region \sqcap area \\ river\_region \dot\sqsubseteq & natural\_region \sqcap area \\ country \doteq & country\_region \sqcap \\ & \forall contains.\neg country\_region \sqcap \\ & \forall overlaps.\neg country\_region \sqcap \\ & \forall inside.\neg country\_region \\ city \doteq & city\_region \sqcap \\ & \exists inside.country\_region \\ lake \dot\sqsubseteq & lake\_region \\ river \doteq & river\_region \sqcap \\ & \forall overlaps.\neg lake\_region \sqcap \\ & \forall contains.\bot \sqcap \\ & \forall inside.\neg lake\_region \\ river\_flowing\_ & \\ into\_a\_lake \doteq & river \sqcap \exists touches.lake\_region\end{array}$$

In [4] more examples on the use of $\mathcal{ALCRP}(\mathcal{D})$ are given, which also demonstrate the influence of spatial reasoning on TBox reasoning (subsumption of concepts). In addition to our previous work, we consider the following spatioterminological *default rules* d_1,d_2 and d_3:

$$\frac{area\ :\ city}{city} \quad \frac{area\ :\ lake}{lake} \quad \frac{area\ :\ country}{country}$$

Below we will show how to use ABoxes instead of concept terms inside the α,β_i and γ of the default rules.

Suppose we have an ABox

$$\{a : country, b : area, (a,b) : contains, (b,a) : inside\}.$$

Closing defaults, i.e. instantiating the defaults d_1,d_2,d_3 over the ABox individuals a and b yields 6 different closed defaults. Now, let us assume α, β and γ have been replaced by the corresponding assertional axioms. We use the notation $d_i(ind)$ to refer to a default that is instantiated with the individual *ind*. Given our 6 closed default rules let us examine the status of each:

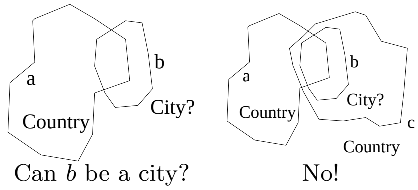

Fig. 1. Subtle inferences due to topological constraints.

- Default $d_1(a)$ cannot be applied because adding a : *city* to the ABox yields a contradiction with a : *country*. The concepts *country_region* and *city_region* are disjoint (due to *large_scale* and ¬*large_scale*).
- Default $d_1(b)$ can be applied. We get an augmented ABox or *extension one*:

$$\{a : country, b : area, b : city, (a, b) : contains, (b, a) : inside\}$$

- Default $d_2(a)$ cannot be applied because adding a : *lake* to the ABox yields a contradiction with a : *country*. A *country* is an *administrative_region* and a *lake* is defined as a *natural_region*, and both are disjoint concepts.
- Default $d_2(b)$ can be applied. Thus, we can get an augmented ABox or *extension two*:

$$\{a : country, b : area, b : lake, (a, b) : contains, (b, a) : inside\}$$

  However, if we have an ABox already augmented by the conclusion of default $d_1(b)$, b : *city*, we cannot apply $d_2(b)$. So, only one of $d_1(b)$ or $d_2(b)$ can be applied, resulting in two different *extensions*.
- Default $d_3(a)$ cannot be applied, because its conclusion is already entailed by the ABox.
- Default $d_3(b)$ cannot be applied even if no other default has been applied before. Adding the default's consequent b : *country* would yield an inconsistent ABox because a is already known to be a *country* and so, among others, a : ∀*contains*.¬*country_region* holds. Because (a, b) : *contains* holds and b : *country* would imply b : *country_region*, the default cannot be applied.

Another subtle inference can be demonstrated by showing that the default $d_1(b)$ cannot be applied to conclude that object b in Figure 1 is a *city*. Trying to do so would result in a constraint b : *city_region* ⊓ ∃*inside*.*country_region*. Therefore, polygon a cannot be the appropriate *country_region* because (b, a) : *overlaps* holds. Due to the exists restriction there exists an implicit individual c which is a *country_region* such that (b, c) : *inside* holds. As can be seen in Figure 1, there is no way to find a spatial arrangement such that b is inside c and c does not overlap with a or does not contain a. Because a is a *country* and, therefore, may not overlap or may not be contained in another *country_region*, there is no way to conclude that b could possibly be a *city*.

In addition, let us assume we would like to be able to conclude that the spatial relationship between a river and a lake is either *ec* (touches) or *dc* (disconnected). These conclusions cannot be expressed with the terminological default rules introduced in [1] because there α, β_i and γ are limited to *concept expressions*. We therefore extended the terminological default rules introduced in [1] by substituting the concept expressions α, β_i and γ by so-called ABox patterns. These ABox patterns are basically ABoxes with placeholders for individuals (written with capital letters). Closing the default rules instantiates the patterns with all possible combinations of individuals yielding closed defaults whose α, β_i and γ are $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes with role assertions only on complex roles:

$$d_4 = \frac{\{X : lake, Y : river\} \ : \ \{(X, Y) : disjoint\}}{\{(X, Y) : disjoint\}},$$

$$d_5 = \frac{\{X : lake, Y : river\} \ : \ \{(X, Y) : touches\}}{\{(X, Y) : touches\}}$$

Closing the patterns, i.e. instantiating X, Y over the ABox $A = \{l : lake, r : river\}$, would yield eight different closed defaults whose whose α, β_i and γ are $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes. As well as allowing variables such as X and Y, one might also be able to refer to specific ABox individuals in the ABox patterns (for instance, the individual "Bodensee").

In the next section we will show that the *consequence problem* is decidable for terminological default theories with default rules containing $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes consisting only of concept assertions and complex role assertions. Since we can always obtain ordinary ABoxes from our ABox patterns by closing them, the consequence problem is decidable for defaults with ABox patterns as well.

4 Computing Extensions

Intuitively, given a closed terminological default theory (A, D) a deductively closed set of consequences of such a theory is referred to as an *extension*. As usual, the exact definition is given by a fixpoint construction. We cite a formal definition taken from [1]. $Th(\Gamma)$ stands for the deductive closure of a set of formulae Γ. In a description logic context Γ is an ABox.

Definition 2. *Let E be a set of closed formulae and (A, D) be a closed default theory. We define $E_0 := A$ and for all $i \geq 0$*

$$E_{i+1} := E_i \cup \{\gamma \mid \alpha : \beta_1, \ldots, \beta_n/\gamma \in D, \; \alpha \in Th(E_i), \; \neg\beta_1, \ldots, \neg\beta_n \notin Th(E)\}.$$

Then, $Th(E)$ is an extension of (A, D) iff

$$Th(E) = \bigcup_{i=0}^{\infty} Th(E_i)$$

Note that, in principle, this definition for an extension $Th(E)$ has a non-constructive nature because in the definition the deductive closure $Th(E)$ is already used in each iteration step. Nevertheless, as we will see below, the definition induces an algorithm for actually computing extensions if the implicit entailment subproblems in the definition are decidable (see also [1]).

In order to be able to infer spatial relations between domain objects, the basic terminological default reasoning approach described in [1] is adapted. The basic idea is that the precondition, the justifications and the consequent of a default can be ABoxes with complex role axioms.

Definition 3. *A* spatioterminological default rule *d (or spatioterminological default for short) has the form $d = \alpha : \beta_1 \ldots \beta_n/\gamma$ where α, β_i and γ are consistent ABoxes whose unfolded versions contain only concept axioms with restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ concept terms and only predicate-based role axioms of the form $(x, y) : \exists(has\_area)(has\_area).P$ with P being an $\mathcal{S}_2$ predicate of arity two. A* spatioterminological default theory *is a tuple (A, D) where D is a set of spatioterminological default rules and A is a consistent and restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox.*

Lemma 1. *A restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox axiom x is logically entailed by a restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox A,*

$$A \models x, \quad \textit{iff} \quad \begin{cases} x = a : C \longrightarrow \neg SAT(A \cup \{a : \neg C\}) \\ x = (a, b) : \exists(u)(v).P \longrightarrow \\ \quad \neg SAT(A \cup \{(a, b) : \exists(u)(v).\overline{P}\}) \wedge \\ \quad \neg SAT(A \cup \{a : \forall u.\top\}) \wedge \\ \quad \neg SAT(A \cup \{b : \forall v.\top\}) \end{cases}$$

$SAT(A)$ decides the ABox consistency problem for an ABox A, and $u = v = has\_area$.

Proof. The first case is the instance checking problem, which is decidable because C is a restricted concept term. The second case is more problematic, because the $\mathcal{ALCRP}(\mathcal{S}_2)$ language does not provide a negation operator for predicate-based role axioms. However, we can check whether $(a, b) : \exists(has\_area)(has\_area).\overline{P} \vee a : \neg\exists has\_area.\textsf{is-region} \vee b : \neg\exists has\_area.\textsf{is-region}$ holds. The NNF of $\neg\exists has\_area.\textsf{is-region}$ is $\exists has\_area.\textsf{is-no-region} \sqcup \forall has\_area.\top$. Since $\exists has\_area.\textsf{is-no-region}$ is inconsistent, the resulting term is $(a, b) : \exists(has\_area)(has\_area).\overline{P} \vee a : \forall has\_area.\top \vee b : \forall has\_area.\top$. Obviously, this is not an $\mathcal{ALCRP}(\mathcal{S}_2)$ ABox. However, $A \cup \{a_1 \vee a_2 \vee \cdots \vee a_n\}$ is inconsistent iff $\forall a_i : A \cup \{a_i\}$ is inconsistent. Note that the predicate name $\overline{P}$ exists because the concrete domain is required to be admissible.

Theorem 1. *The consequence problem for a spatioterminological default theory (A, D) is decidable.*

Proof. Considering the sound and complete tableaux calculus for deciding the consistency of restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes, $x \in Th(\Gamma)$ iff $\Gamma \models x$. Thus, instead of taking $Th(E)$ we can view the ABox E as a representative for an extension. The fixpoint construction in Definition 2 can be used as a tester for determining whether a given ABox E really *is* an extension of a default theory (A, D). Since each extension E is an ABox having the form $A \cup \{\gamma \mid \alpha : \beta_1 \ldots \beta_n/\gamma \in D'\}$ for a set of so-called *generating defaults* $D' \subseteq D$, we can simply check for each element E of $\{A \cup X \mid X \in 2^{\{\gamma \mid \alpha:\beta_1\ldots\beta_n/\gamma \in D\}}\}$ whether it is an extension or not. The following inference problems need to be decided:

1. $\alpha \in Th(E_i)$: This can be easily tested by checking whether $E_i \models \alpha$ where $\alpha = \{a_1, a_2, \ldots, a_n\}$. We can decide this *ABox entailment problem* iff we can decide whether each assertional axiom a_i follows from A, i.e. $\forall a_i \in E_i : A \models a_i$. This can be decided according to Lemma 1 because the elements of α are restricted to be concept axioms or predicate-based role axioms.
2. $\neg\beta_i \notin Th(E_i)$: This can be checked by testing whether $E \not\models \neg\beta_i$. More generally, $A \not\models \neg B$, where $B = \{b_1, b_2, \ldots, b_n\}$ iff $\forall b_i \in B : A \not\models \neg b_i$. However, $A \not\models \neg b_i$ iff $A \cup \{b_i\}$ is consistent. The ABox consistency problem for restricted $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes is decidable.
3. $Th(E) = \bigcup_{i=0}^{\infty} Th(E_i)$: The fixpoint can be constructed in a finite number of steps because we consider only a finite number of defaults. In principle, we have to decide the *ABox equivalence problem*. An ABox A_1 is equivalent to an ABox A_2, $A_1 \equiv A_2$ iff $A_1 \models A_2$ and $A_2 \models A_1$, i.e. the ABox equivalence problem can be reduced to two ABox entailment problems. Unfortunately, considering $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes there might not only be concept axioms and predicate-based role axioms in A_1 or A_2 but also role axioms of the form $x = (a, b) : R$ or $x = (a, b) : f$ or $x = (x_1, \ldots, x_n) : P$ where R is a role name, f is a feature and P is an $\mathcal{S}_2$ predicate of arity n. In

this case Lemma 1 is not applicable. However, both $A_1(=E)$ and $A_2(=E_n)$ are constructed on the basis of A, that is, we have to decide whether two ABoxes of the form $A_1 = A \cup \Gamma_1$ and $A_2 = A \cup \Gamma_2$ are equivalent, where $\Gamma_i \subseteq \{\gamma \,|\, \alpha : \beta_1 \dots \beta_n/\gamma \in D\}$. Obviously, $(A \cup \Gamma_1) \equiv (A \cup \Gamma_2)$ iff $A \cup \Gamma_1 \models \Gamma_2$ and $A \cup \Gamma_2 \models \Gamma_1$. Since both Γ_1 and Γ_2 contain only concept axioms and predicate-based role axioms, Lemma 1 is applicable.

In [1] another algorithm is discussed for computing extensions. This algorithm seems to be more efficient in the average case. There is a strong conjecture that the algorithm is also applicable in the $\mathcal{ALCRP}(\mathcal{S}_2)$ context. Furthermore, it can easily be seen that the results for spatioterminological default theories wrt. $\mathcal{ALCRP}(\mathcal{S}_2)$ can be extended to $\mathcal{ALCRP}(\mathcal{D})$ as well.

4.1 A Note on Terminological Default Reasoning with Specificity

Consider the world description

$$A = \{r : river\_flowing\_into\_a\_lake, l : lake\}.$$

Since it is already known that r is a *river_flowing_into_a_lake* and not only a *lake*, we would like to conclude that the lake l in A should be *the* lake. That is, the complex role assertion $(l, r) : touches$ should be added:

$$d_6 = \frac{\{X : lake, Y : river\_flowing\_into\_a\_lake\} \ : \ \{(X, Y) : touches\}}{\{(X, Y) : touches\}}$$

In the case of d_6, we would like to render the application of d_4 and d_5 *invalid*, because they are "less specific" than d_6 (even if d_5 yields the same conclusion, *touches*).

A default d_a is said to be more specific than d_b, $d_a \prec d_b$ iff $(\alpha(d_a) \models \alpha(d_b)) \wedge (\alpha(d_b) \not\models \alpha(d_a))$ where $\alpha(D)$ denotes the precondition of the default D. Algorithms for computing the so-called *S-extensions* (S for specificity) have already been developed by Baader and Hollunder [2]. There is a strong conjecture that these algorithms can be applied in our $\mathcal{ALCRP}(\mathcal{S}_2)$ context as well. In contrast, the ordinary extensions are called *R-extensions* (R for Reiter). In our example, we would get two different R-extensions, but only one S-extension containing the ABox axiom $(r, l) : touches$. The other *R-extension* containing $(r, l) : disjoint$ could not be derived, since only the most specific active defaults are applied when computing S-extensions. This would render the application of d_4 and d_5 impossible because d_6 is also active and more specific than both d_4 and d_5.

5 Conclusion

To the best of our knowledge we have proposed a first theory for spatioterminological default reasoning. Our spatioterminological default approach extends previous work done in [3, 4]. The new contributions to [1] are: As a base language, the expressive spatioterminological description logic $\mathcal{ALCRP}(\mathcal{S}_2)$ is used. Allowing not only concept terms as formulae occurring inside default rules but also special $\mathcal{ALCRP}(\mathcal{S}_2)$ ABoxes with complex role assertions is necessary from an application-oriented point of view but imposes a number of theoretical problems. We have shown that the possible extensions of a closed $\mathcal{ALCRP}(\mathcal{S}_2)$ spatioterminological default theory can be effectively computed provided the ABoxes used in the default rules contain only (restricted) concept and complex role assertions.

An implementation of $\mathcal{ALCRP}(\mathcal{D})$ is described in [5]. With the implementation of the $\mathcal{ALCRP}(\mathcal{D})$ default reasoning substrate, an implementation of an $\mathcal{ALCRP}(\mathcal{D})$ TBox and ABox management system as well as an RCC-8 relation network consistency checker is also available for research purposes. Qualitatively speaking, tests with the current implementation indicate that for small problems with few ABox assertions, results can be expected in a reasonable time but runtimes dramatically increase when more than only a few individuals are involved. Optimizations can be achieved, for instance, if queries for the extensions are considered [6].

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