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Spatioterminological Reasoning: Subsumption Based on Geometrical Inferences

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Abstract: *This paper presents a theoretical basis for terminological reasoning about objects and their qualitative spatial relationships. In contrast to existing work, which mainly focuses on reasoning about qualitative spatial relations alone, we integrate quantitative and qualitative information with terminological reasoning. This theory is motivated as basis for knowledge representation and query processing for instance in the domain of deductive geographic information systems.*

Keywords: Qualitative spatial reasoning, terminological reasoning.

1 Introduction

The combination of formal conceptual and spatial reasoning serves as a theoretical basis for knowledge representation in domains such as geographical information systems (GIS) and can be used to solve important application problems. For instance, spatioterminological inferences can be applied to interpretation of map databases [6] and to spatial query processing [9].

Our treatment of spatial reasoning is based on Egenhofer's set of topological relations [3] while the terminological reasoning part is based on description logic (DL) theory. In contrast to our earlier work presented in [7], [5] and [8] where topological relations are used as primitives in the sense of logic, we extend the treatment of topological relations with respect to conceptual reasoning by interpreting their semantic definition and by demonstrating their influence on automatic concept classification.

2 Integrating Spatial and Terminological Reasoning

This section introduces a space box (SBox) reasoner which implements inference services over spatial regions and concept terms. The SBox reasoner complements the usual DL reasoning facilities concerning the TBox and Abox. We analyze current possibilities to integrate the SBox into the CLASSIC system. The integration is based on a recent proposal [2] that extended the theory behind the CLASSIC DL for coping with external domains.

The fundamental idea of our SBox extension is the treatment of spatial regions as subsets of $\mathbb{R}^2$ represented by polygons and to define so-called *spatial* subsumption between polygons with respect to the relation g_contains (or g_inside, see Figure 1). Basically, spatial subsumption can be reduced to the polygon inclusion and intersection problem. The restriction to polygons is motivated by computational geometry offering efficient algorithms for polygon inclusion and intersection. As we will see, additional spatial inferences must be supported (reasoning with spatial relations) because not all polygons must necessarily be given as constants.

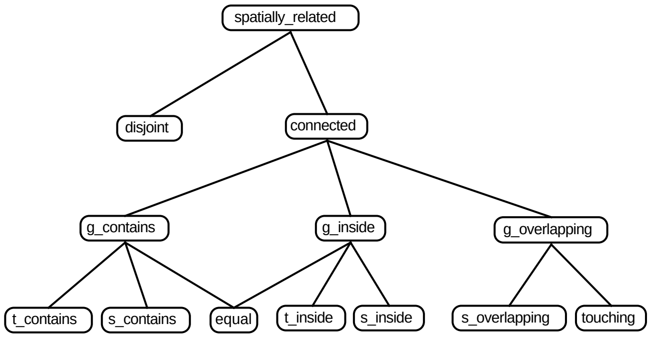

Figure 1: Subsumption hierarchy of spatial relations.

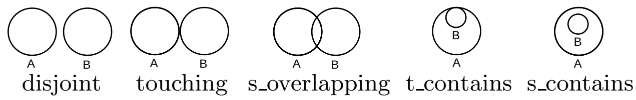

Figure 2: Elementary spatial relations between A and B.

2.1 Spatial Relations

In a similar way as [4] we define 13 binary topological relations that are organized in a subsumption hierarchy (see Figure 1). The leaves of this hierarchy represent eight mutually exclusive relations (*elementary* relations) that are equivalent to the set of relations defined by Egenhofer [3]. The non-elementary relations are defined by a disjunction of relations represented as direct descendants of the corresponding nodes. Figure 2 illustrates five elementary relations (the inverses and the relation 'equal' have been omitted). Due to lack of space we refer to [6] for a formal definition of these relations.

2.2 New Language Constructs as External Concept Expressions

In order to support spatial inferences, we introduce new concept constructors based on these spatial relations. Our semantics assumes that each domain object is associated with its spatial representation (i.e. a polygon) via a predefined attribute has_area (see Figure 3). Spatial concepts for the external domain are denoted as $\mathsf{sr_p}$

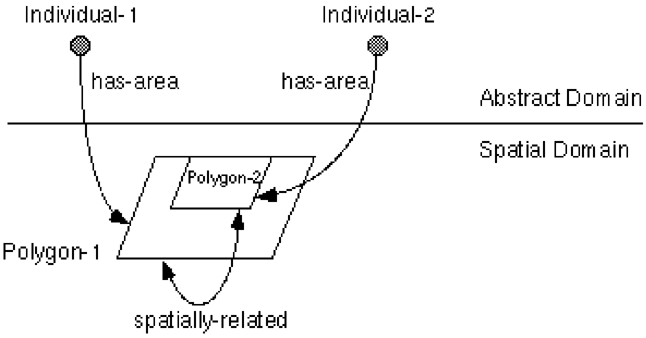

Figure 3: Relationship between abstract and concrete objects.

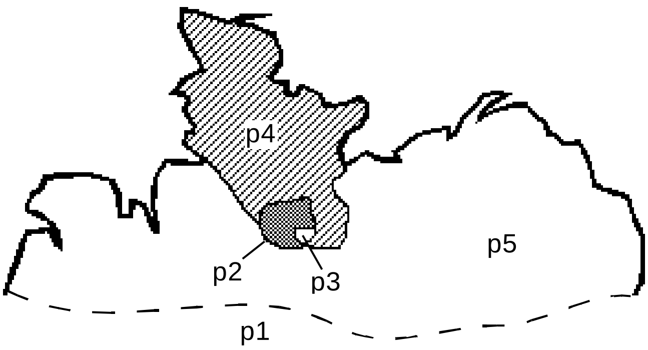

Figure 4: A sketch of the northern part of Germany with polygons for Germany (p_1), Northern Germany (p_5), the federal states Schleswig-Holstein (p_4) and Hamburg (p_2) as well as a small district of Hamburg (p_3). Polygon p_3 is assumed to be inside p_2 but p_2 is not inside p_4.

where sr is a relation from Figure 1 and p is a polygon constant. The integration of the abstract and the external, spatial domain is realized with ∀-restrictions on the fillers of the attribute has_area (see below). We extend the range of the DL interpretation function ξ to the set of polygons $\mathcal{P}$ where each polygon $\mathsf{p} \in \mathcal{P}$ defines a subset of $\mathbb{R}^2$. The operator $\mathsf{sr_p}$ has the following semantics.

$$\xi[\mathsf{sr_p}] = \{x \mid (x, \mathsf{p}) \in \xi[\mathsf{sr}])\} \text{ with } \xi[\mathsf{sr}] \subseteq \mathcal{P} \times \mathcal{P}$$

For instance, we use the constructor $\mathsf{g\_inside_p}$ to define concepts for a region in Northern Germany, for a district of the city of Hamburg etc. (HH is part of the car license number for Hamburg).

northern_german_region $\doteq$ (∀ has_area $\mathsf{g\_inside_{p_5}}$)

district_of_hh $\doteq$ (∀ has_area $\mathsf{g\_inside_{p_2}}$)

The corresponding spatial constellation is illustrated in Figure 4. The construct (∀ has_area $\mathsf{g\_inside_{p_5}}$) subsumes every region of Northern Germany whose associated polygon is g_inside of $\mathsf{p_5}$. With the operator $\mathsf{equal_p}$, we define concepts for the federal states Hamburg and Schleswig-Holstein.

federal_state_hh $\doteq$ (∀ has_area $\mathsf{equal_{p_2}}$)

federal_state_sh $\doteq$ (∀ has_area $\mathsf{equal_{p_4}}$)

For instance, federal_state_hh is subsumed by northern_german_region since $\xi[\mathsf{equal_{p_2}}] \subseteq \xi[\mathsf{g\_inside_{p_5}}]$. We like to exphasize that $\mathsf{equal_{p_2}}$ cannot subsume other spatial concepts. Algorithms for deciding subsumption between $\mathsf{sr_p}$ concepts are explained in Section 2.3.

In many cases, restrictions about spatial relations will have to be combined with additional restrictions. For example, how can we define a concept that describes a district of Hamburg that touches the federal state Hamburg from the inside? This requires some kind of qualified existential quantification. Thus, we propose the concept-forming operator (◯ sr c) with the following semantics (let sr denote a spatial relation and c a concept term):

$$\begin{aligned}\xi[(\bigcirc\ \mathsf{sr}\ \mathsf{c})] = \{x \mid \exists y_1, y_2, z :\ & (x, y_1) \in \xi[\mathsf{has\_area}], \\ & (z, y_2) \in \xi[\mathsf{has\_area}], \\ & (y_1, y_2) \in \xi[\mathsf{sr}],\ x \neq z,\ z \in \xi[\mathsf{c}]\}\end{aligned}$$

With this new operator we define the following two concepts. It can be proven that hh_border_district_to_sh is subsumed by hh_border_district.

hh_border_district $\doteq$
district_of_hh ⊓ (◯ t_inside federal_state_hh)

hh_border_district_to_sh $\doteq$
district_of_hh ⊓ (◯ touching federal_state_sh) ⊓ (◯ spatially_related federal_state_hh)

In the next section we discuss how inferences about the new concept-forming terms can be realized with the CLASSIC extension interface.

2.3 Extending the CLASSIC Description Logic

Borgida et al. [2] defined the following set of functions for integrating a new concept-forming operator **K** into the CLASSIC description logic system. These functions are declared to the CLASSIC inference engine and are automatically called during subsumption proofs when required.

Normalization

In addition to syntax checking a normalization function for each term constructor **K** is required (in the following, the constructor pattern **K** is written in square brackets). As part of the normalization phase, all defined concepts are replaced by their definition.

- $\mathsf{NormalizeTerm[sr_p](sr_p) = sr_{NormalizePolygon(p)}}$
- NormalizeTerm[(◯ sr c)]((◯ sr c)) = (◯ sr NormalizeTerm(c))

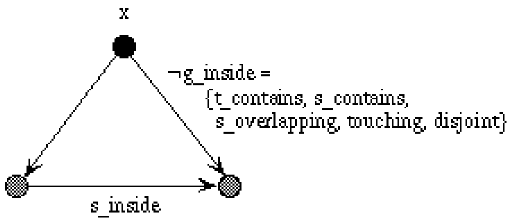

Figure 5: Constraint network for computing the subsumption relation between two concepts. The constraint system is inconsistent.

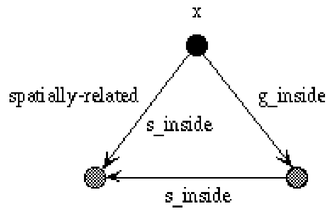

Figure 6: Example for a term that is implied by a conjunction of spatial host concepts.

Subsumption

Structural subsumption has to deal with terms that either contain external predicate terms (see [2]) or are equal to a predicate term. In our case, an external predicate term (used as a concept) refers to a polygon p explicitly given in $\mathsf{sr_p}$. In CLASSIC's terminology, an external predicate term $\mathsf{sr_p}$ is called a *host concept*. Host concepts may not be combined with abstract concepts (e.g. in conjunctions).

- StructuralSubsumes?[$\mathsf{sr_r}$]($\mathsf{sr^1_p}$,$\mathsf{sr^2_q}$) returns true iff $\forall \mathsf{x} \in \mathcal{P} : \mathsf{sr^1_p(x)} \Leftarrow \mathsf{sr^2_q(x)}$. In other words: $\exists \mathsf{x} \in \mathcal{P} : \neg \mathsf{sr^1(x,p)} \wedge \mathsf{sr^2(x,q)}$ must be inconsistent.

Thus, in order to check whether $\mathsf{g\_inside_{p_5}}$ subsumes $\mathsf{equal_{p_2}}$ (see above) the constraint system presented in Figure 5 must be solved. Before well-known algorithms for solving spatial constraint systems (based on Egenhofer's composition table [3]) can be applied, restrictions concerning "concrete" polygon constants must be computed. In the example shown in Figure 5, $\mathsf{p_2}$ (Hamburg) is known to be s_inside $\mathsf{p_5}$ (Northern Germany). The constraint system is obviously inconsistent because equal composed with s_inside is defined to be s_inside. However, ¬g_inside does not contain s_inside (see also Figure 1). Thus, $\mathsf{equal_{p_2}}$ is subsumed by $\mathsf{g\_inside_{p_5}}$. Grigni et al. have emphasized [4] that constraint systems that are (relationally) consistent must not necessarily lead to situation that are *realizable* in the plane. Thus, an additional planarity test must be added (see also [11]). For other concept-forming operators similar techniques can be applied.

- StructuralSubsumes?[(◯ sr c)]((◯ $\mathsf{sr^1}$ c1),(◯ $\mathsf{sr^2}$ c2)) returns true iff
  - c2 ⊑ c1 and
  - $\exists \mathsf{x,y,z} \in \mathcal{P} : \neg \mathsf{sr^1(x,y)} \wedge \mathsf{sr^2(x,z)}$ is inconsistent.

In order to compute whether a concept term based on the constructor **K** subsumes a general concept term which is constructed with other concept constructors, we have to check whether the general concept implies the concept based on **K**.

- Subsumes?[$\mathsf{sr_p}$]($\mathsf{sr_p}$,normalizedHostConcept) returns true iff the conjunction normalizedHostConcept ⊓ ¬$\mathsf{sr_p}$ is inconsistent. From normalizedHostConcept we only consider the predicate terms $\mathsf{sr^i_{p_j}}$. This is a generalization of StructuralSubsumes?. For instance, $\mathsf{s\_inside_{p_1}}$ is subsumed by the conjunction $\mathsf{spatially\_related_{p_1}} \sqcap \mathsf{g\_inside_{p_2}}$ (see Figure 6). Due to the constraint propagation process, spatially_related is restricted to s_inside because, according to Figure 4, $\mathsf{p_2}$ is s_inside $\mathsf{p_1}$. If we claim that $\neg\mathsf{s\_inside_{p_1}(x)}$ holds, the constraint system becomes inconsistent.

In order to check whether hh_border_district is implied by hh_border_district_to_sh it must be shown that the conjuction district_of_hh ⊓ (◯ touching federal_state_sh) ⊓ (◯ spatially_related federal_state_hh) (or its normalized form) implies (◯ t_inside federal_state_hh). To be able to prove this implication, a decision procedure for the pattern **K** = (◯ sr c) must be declared with CLASSIC's extension interface.

- Subsumes?[(◯ sr c)]((◯ sr c),normalizedConcept) returns true iff (◯ sr c) is implied by normalizedConcept. We have to extract from normalizedConcept every term of the form (◯ sr c) or (∀ has_area $\mathsf{sr_p}$) and to combine them as a conjunction SC and check whether $\exists \mathsf{x} : \mathsf{SC(x)} \wedge \neg$ (◯ sr c)(x) is inconsistent.

  The decision procedure will be explained with the example from above. We start with SC = (∀ has_area $\mathsf{g\_inside_{p_2}}$) ⊓ (◯ touching federal_state_sh) ⊓ (◯ spatially_related federal_state_hh) and want to derive that (◯ t_inside federal_state_hh) is implied. From the concept terms given with SC we construct a graph. In Figure 7 an individual x has been generated. For this individual x all role fillers of has_area are g_inside $\mathsf{p_2}$ because district_of_hh holds. Since has_area is an attribute, a single filler can be generated as a representative ($\mathsf{q_2}$, see Figure 7). The constraint $\mathsf{g\_inside(q_2, p_2)}$ is added. The other two terms are treated as follows. Due to the exists semantics of the circle operator, two additional individuals y and z are generated, together with their associated geometrical representations q_3 and q_1, respectively. From the circle terms we know the constraints $\mathsf{spatially\_related}(q_2, q_1)$ and $\mathsf{touching}(q_2, q_3)$. Since z is subsumed by federal_state_hh, $\mathsf{equal}(q_1, p_2)$ also holds (see the structure created in Figure 7). Furthermore, $\mathsf{equal}(q_3, p_4)$ holds, because y is subsumed by federal_state_sh.

In Figure 8, implicit relations between spatial objects have been added and the constraints have been solved. Obviously, because q_1 is equal to p_2, $\mathsf{g\_inside}(q_2, q_1)$ also holds. Since p_2 is touching p_4 (see Figure 4), the relation between q_2 and q_1 is further restricted to t_inside. Now, in order to check whether (◯ t_inside federal_state_hh) is subsumed, the resulting graph structure is traversed (starting from x and following has_area), i.e. direct paths to the generated objects are examined. In our example structure, there are two (direct) paths to new individuals (z is reached via t_inside and y is reached via touching). The concepts c_i of the individuals at the end of each of these paths are "matched" against the concept term c of the (◯ sr c) term in question. If there exists a path with relation r to an individual whose c_i is subsumed by c with r being equal to or a subrelation of sr, then the (◯ sr c) term is implied by SC. This is indeed the case for (◯ t_inside federal_state_hh).

In a similar way as for sr_p we declare a subsumption checker for $\mathbf{K} = (\forall$ has_area $sr_p)$.

- Subsumes?[($\forall$ r c)](($\forall$ has_area sr^1_p),normalizedConcept) returns true iff normalizedConcept contains (◯ sr^2_p c0), c0 implies ($\forall$ has_area sr^3_p), and $\exists x, y : \neg sr^1(y, p) \wedge sr^2(y, x) \wedge sr^3(x, p)$ is inconsistent. ($\forall$ has_area equal_{p_2}) is also implied by a (fills has_area p_2) term because has_area is an attribute. Note that although CLASSIC adopts a non-standard semantics for fills, this is not relevant for host concepts since properties of host individuals cannot be changed by concept terms.

Conjoining Concept Terms

The functions for conjoining concept terms and consistency checking are similar to the subsumption functions given above. Implied terms (see above) must also be considered. In some cases, conjunctions can be simplified. For brevity, we do not discuss conjoin functions in detail in this paper.

3 Related Work

Concerning description logic theory, another general technique for integrating external domains into DLs is the 'concrete domain' approach [1; 10]. For instance,

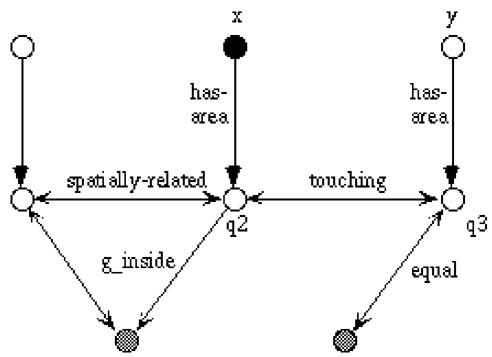

Figure 7: Initial structure used for deriving the subsumption relation beween hh_border_district and hh_border_district_to_sh. For symmetric relations the arrows point in both directions.

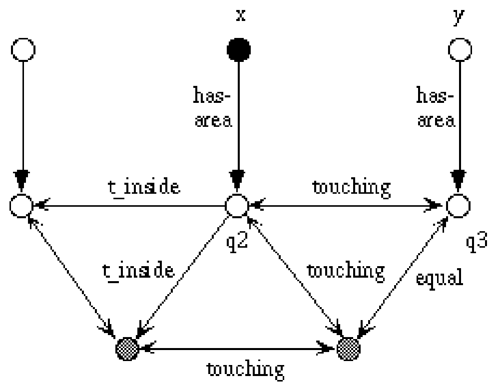

Figure 8: Structure from Figure 7 with implicit relations added and constraints propagated. Irrelevant relations have been omitted for clarity.

$\mathcal{ALC}(\mathcal{D})$ provides a simple interface for external domains basically requiring that the satisfiability of finite conjunctions of concrete predicates be decidable. However, this approach can only define concepts with concrete predicates that depend on information available from attribute chains starting with this concept. Spatial relations cannot be adequately defined with the operators and primitive roles offered by $\mathcal{ALC}(\mathcal{D})$. Another solution might be the new role-forming operator of $\mathcal{ALCRP}(\mathcal{D})$ as proposed in [11]. Then, the term (◯ sr c) could be replaced by ($\exists$ sr(has_area)(has_area) c). However, the satisfiability problem for $\mathcal{ALCRP}(\mathcal{D})$ has shown to be undecidable (see [11]).

Grigni et al. [4] study the computational problems in developing an inference system for checking the satisfiability of (conjunctive) combinations of spatial relations. They point out that in topological inferencing the aspects of relational consistency and planarity interact in rather complex ways. They showed that besides the relational consistency problem a planarity problem has to be solved when areas are assumed to be disjoint. With this additional restriction, in many cases the complexity of the satisfiability problem becomes NP-hard.

4 Conclusions

In this paper, we have developed a DL formalization of space with two separate domains: the *abstract* and the *space domain*. The abstract domain is used to represent terminological knowledge about spatial domains on an abstract logical level. The space domain extends the abstract domain and allows access to efficient reasoning algorithms (e.g. computational geometry, spatial indexing) for concrete spatial regions (e.g. polygons in map databases). We have demonstrated that, on the one hand, topological relations directly influence the kind of conceptual or terminological knowledge that can (and must) be derived by a formal inference engine. On the other hand, assertions about concepts restrict the set of possible spatial relations between different individuals.

Due to CLASSIC's complex extension scheme for external domains, the integration of our proposed operators into CLASSIC appears to be less elegant than, for instance, the $\mathcal{ALC}(\mathcal{D})$ approach. The high complexity is caused by delegating to the user the full responsibility for capturing all (hidden) inferences associated with an external domain. However, the spatial inference rules presented in this paper indicate that CLASSIC's DL extended by our operators still remains decidable. We do not support spatial relations in $\forall$-terms and only a limited form of exists-in restrictions for spatial relations can be defined.

Acknowledgments

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