

## SBox: A Qualitative Spatial Reasoner —Progress Report—

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### Abstract

*This paper presents a method for reasoning about spatial objects and their qualitative spatial relationships (e.g. touches, overlaps etc.) on the basis of a description logic framework. We apply this method to the domain of deductive geographic information systems. In contrast to existing work, which mainly focuses on reasoning about qualitative spatial relations alone, we integrate quantitative and qualitative information with terminological reasoning by extending description logics with a space box reasoner which is inspired by an extension to description logics called “concrete domains.” With the space box reasoner presented in this paper it is possible to combine qualitative spatial reasoning and description logic classification processes.*

**Keywords**— Qualitative spatial reasoning, description logics, deductive geographical information systems.

### 1 Introduction

Qualitative relations play an important role in formal reasoning systems. We emphasize that inferences about spatial relations should not be considered in isolation but should be integrated with formal inferences about concepts (e.g. automatic consistency checking and classification). The semantics of qualitative relations should be grounded in a quantitative representation of spatial data. In our opinion, the abstractions provided by qualitative spatial relations can be interpreted as an interface from a conceptual model about the world to quantitative spatial data representing spatial information about domain objects.

The combination of conceptual and spatial inference services can be used to solve important application problems. Continuing our work presented in [10] we show how terminological inferences with spatial relations can be used for image interpretation. The characteristic of these problems is that it is often very difficult to describe a fixed algorithm that defines an exact sequence of “interpretation steps” because several different “cues” have to be integrated. In other words: the solution must be computed by adequately integrating partial information about domain objects.

The information about objects is given by conceptual background knowledge, the image itself and different kinds of intermediate interpretation results. According to the work of Schröder and Neumann [19] who are inspired by the MAPSEE approach [17], image interpretation can be defined as a (re-)construction process of a specific possible world that is consistent with the given knowledge (see also Section 5).

In this paper, we consider a map interpretation problem and demonstrate how conceptual background knowledge can be exploited for image interpretation tasks. As an example, a subsection of a map from the city of Hamburg is shown in Figure 1. In a geographical information system, queries like “search for a living area in a border district with recreation areas” might be defined. We assume that the necessary data are automatically gathered using image interpretation techniques. Note that in our setting image interpretation starts with vector data, i.e. areas are defined by polygons (see the emphasized polygon in the center of Figure 1). Polygons from the image data are already annotated with labels like “living-area”, “ordinary-road” etc. In order to interpret the image, different kinds of world knowledge are required. For instance, with background knowledge one can infer that the large number 7434 in the upper right corner of Figure 1 cannot be a zip code nor can it describe the height of a mountain (not in Northern Germany). The required inference steps can be formalized by combining spatial and terminological reasoning.

The spatial part of the theory is based on Egenhofer’s set of topological relations. In contrast to [10] and [9] where topological relations are used as primitives in the sense of logic (i.e. they are semantically uninterpreted), we extend the treatment of topological relations by interpreting their semantic definition with respect to concept entailment (cf. the notion of subsumption: one concept is more general than another) and demonstrate their influence on automatic concept classification.

Thus, the theory presented in this paper allows to detect both inconsistencies and implicit information in formal conceptual models for spatial domain objects. On the one hand, it can be shown that concept definitions and subsumption (or inheritance) relations restrict the set of possible relations between domain objects. On the other hand, definitions about topolog-

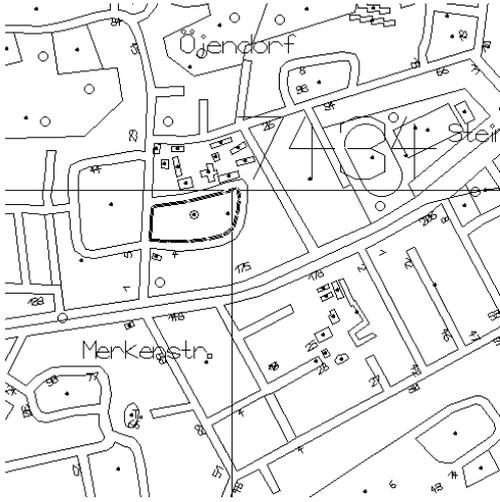


Figure 1: Subsection of Öjendorf, a district of the city of Hamburg.

ical relations might define implicit subsumption relationships which have to be automatically detected to capture all kinds of possible inferences that are sanctioned by the semantics of the representation formalism.

The major contribution of this paper is the incorporation of characteristics of space into the semantics of the inference system. The main idea is to treat a region as a set of points and to extend the subsumption relationship between concepts to subsumption between spatial regions. A region  $R_1$  can be defined to subsume another region  $R_2$  when  $R_1$  “contains”  $R_2$  (see Section 2 for a formal definition of spatial relations). Basically, for spatial subsumption, the same set-inclusion semantics as for concept languages is used. For our application we consider spatial point sets defined by polygons. Qualitative relations between two dimensional areas are defined by topological relations between polygons.

## 2 Qualitative Modeling

The previous section motivated the formalization (qualitative modeling) of space with the help of conceptual and spatial inference services. This section introduces the formal tools used for qualitative modeling. We define spatial regions and their qualitative relationships and combine them with a description logic framework extended by a space box reasoner.

### 2.1 Objects and their Spatial Relationships

The definition of basic geometric objects usually relies on topology which is itself a basis for defining relationships between objects. In the following we assume the usual concepts of point-set topology with open and closed sets [20]. The *interior* of a set  $\lambda_i$  (denoted by  $\lambda_i^\circ$ ) is the union of all open sets in  $\lambda_i$ . The *closure*

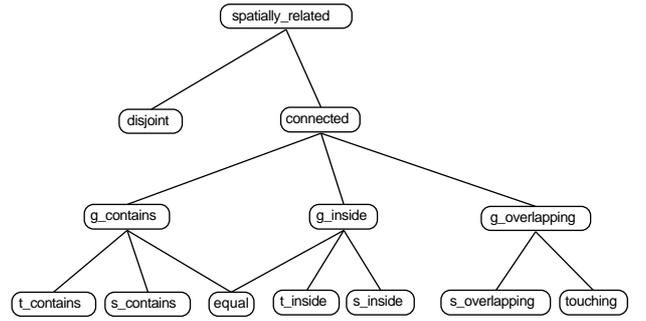


Figure 2: Hierarchy of spatial relations



Figure 3: Spatial relations between A and B

of  $\lambda_i$  (denoted by  $\overline{\lambda_i}$ ) is the intersection of all closed sets containing  $\lambda_i$ . The *complement* of  $\lambda_i$  (denoted by  $\lambda_i^{-1}$ ) with respect to the embedding space  $\mathfrak{R}^n$  is the set of all points of  $\mathfrak{R}^n$  not contained in  $\lambda_i$ . The *boundary* of  $\lambda_i$  (denoted by  $\partial\lambda_i$ ) is the intersection of the closure of  $\lambda_i$  and the closure of the complement of  $\lambda_i$ .

The following restrictions apply to every pair of sets. (1)  $\lambda_i, \lambda_j$  be  $n$ -dimensional and  $\lambda_i, \lambda_j \subset \mathfrak{R}^n$ , (2)  $\lambda_i, \lambda_j \neq \emptyset$ , (3) all boundaries, interiors, and complements are connected, and (4)  $\lambda_i = \overline{\lambda_i^\circ}$  and  $\lambda_j = \overline{\lambda_j^\circ}$ .

Using these definitions we can define 13 binary topological relations that are organized in a subsumption hierarchy (see Figure 2). The leaves of this graph represent eight mutually exclusive relations that cover all possible cases with respect to the restrictions mentioned above. The eight relations are also referred to as *elementary relations*. The elementary relations are equivalent to the set of eight relations defined by Egenhofer [7] and others [16, 6]. Figure 3 illustrates five of these eight relations (the inverses and the relation equal are omitted). The 13 relations are defined as follows:

- **spatially\_related**: Two objects have a *spatial relationship* between each other. This relation is defined as the disjunction of its two mutually exclusive subrelations *disjoint* and *connected*.  

$$\text{spatially\_related}(\lambda_1, \lambda_2) \equiv \text{disjoint}(\lambda_1, \lambda_2) \vee \text{connected}(\lambda_1, \lambda_2)$$
- **disjoint**: Two objects are *disjoint* if their intersection is empty; *disjoint* is symmetric.  

$$\text{disjoint}(\lambda_1, \lambda_2) \equiv \lambda_1 \cap \lambda_2 = \emptyset$$
- **connected**: Two objects are *connected* if their intersection is non-empty; *connected* is symmetric.  

$$\text{connected}(\lambda_1, \lambda_2) \equiv \lambda_1 \cap \lambda_2 \neq \emptyset$$

- **g\_overlapping**: Two objects are *generally overlapping*. This relation is defined as the disjunction of its two mutually exclusive subrelations **touching** and **s\_overlapping**; **g\_overlapping** is symmetric.

$$\text{g\_overlapping}(\lambda_1, \lambda_2) \equiv \text{touching}(\lambda_1, \lambda_2) \vee \text{s\_overlapping}(\lambda_1, \lambda_2)$$

- **touching**: Two objects are *touching* if only their boundaries are intersecting; **touching** is symmetric.

$$\text{touching}(\lambda_1, \lambda_2) \equiv \text{connected}(\lambda_1, \lambda_2) \wedge (\lambda_1^o \cap \lambda_2^o = \emptyset)$$

- **s\_overlapping**: Two objects are *strictly overlapping* if they are connected and their intersection is not equal to either of them; **s\_overlapping** is symmetric.

$$\text{s\_overlapping}(\lambda_1, \lambda_2) \equiv \text{connected}(\lambda_1, \lambda_2) \wedge (\lambda_1 \cap \lambda_2 \neq \lambda_1) \wedge (\lambda_1 \cap \lambda_2 \neq \lambda_2) \wedge (\lambda_1^o \cap \lambda_2^o \neq \emptyset)$$

- **g\_contains/g\_inside**: An object  $\lambda_1$  *generally contains* an object  $\lambda_2$ . This relation is defined as the disjunction of its three mutually exclusive subrelations **equal**, **t\_contains**, and **s\_contains**; **g\_inside** is the inverse of **g\_contains**; **g\_contains** and **g\_inside** are reflexive, antisymmetric, and transitive.

$$\text{g\_contains}(\lambda_1, \lambda_2) \equiv \text{t\_contains}(\lambda_1, \lambda_2) \vee \text{s\_contains}(\lambda_1, \lambda_2) \vee \text{equal}(\lambda_1, \lambda_2)$$

- **equal**: The relation **equal** is symmetric and transitive.

$$\text{equal}(\lambda_1, \lambda_2) \equiv \lambda_1 = \lambda_2$$

- **t\_contains/t\_inside**: An object  $\lambda_1$  *tangentially contains* an object  $\lambda_2$  if their intersection is equal to  $\lambda_2$  and the intersection of their boundaries is non-empty; the inverse of **t\_contains** is **t\_inside**; **t\_contains** and **t\_inside** are asymmetric.

$$\text{t\_contains}(\lambda_1, \lambda_2) \equiv (\lambda_1 \cap \lambda_2 = \lambda_2) \wedge (\lambda_1 \cap \lambda_2^{-1} \neq \emptyset) \wedge (\partial\lambda_1 \cap \partial\lambda_2 \neq \emptyset)$$

- **s\_contains/s\_inside**: An object  $\lambda_1$  *strictly contains* an object  $\lambda_2$  if their intersection is equal to  $\lambda_2$  and only the interiors of their regions intersect; the inverse of **s\_contains** is **s\_inside**; **s\_contains** and **s\_inside** are asymmetric and transitive.

$$\text{s\_contains}(\lambda_1, \lambda_2) \equiv (\lambda_1 \cap \lambda_2 = \lambda_2) \wedge (\lambda_1 \cap \lambda_2^{-1} \neq \emptyset) \wedge (\partial\lambda_1 \cap \partial\lambda_2 = \emptyset)$$

## 2.2 Description Logic

We model terminological knowledge about our GIS domain using description logic (DL) theory that has also been proven as a useful formalism for modeling in technical domains (see e.g. [22] and [14] for example applications). In addition, the formal properties of description logics have been extensively studied (see e.g. [15] and [21]).

The following sections give a brief introduction to some aspects of DL theory. We do not attempt to give a thorough overview and formal account of DL theory. However, we try to summarize the notions important for this paper and refer to [5, 3, 11] for more complete information about description logic theory.

### 2.2.1 DL: The Abstract Domain

In a DL a factual world consists of named individuals and their relationships that are asserted through binary relations. Hierarchical descriptions about sets of individuals form the terminological knowledge. Descriptions (or terms) about sets of individuals are called *concepts* and binary relations are called *roles*. Descriptions consist of identifiers denoting concepts, roles, and individuals, and of description constructors. Concepts or roles may be either *primitive* or *defined*. A specification of a primitive concept is denoted with the declaration operator ‘ $\sqsubseteq$ ’ and represents membership conditions that are necessary but *not* sufficient. The specification of a defined concept is denoted by ‘ $\doteq$ ’ and represents conditions that are both necessary *and* sufficient. For any individual  $x$  the set  $\{y|r(x, y)\}$  is called the set of *fillers* of role  $r$ .

Concept specifications may consist of concept terms and concept names. Unary (e.g.  $\neg$ ) are used as modifiers and binary operators (e.g.  $\wedge$ ,  $\vee$ ) are used as connectives. A concept term can also be given as a restriction for role fillers. *Value restrictions* constrain the range of roles and allow only fillers that are individuals of a specific concept (e.g.  $(\forall \text{has\_father male})$ ). *Number restrictions* specify the maximum or minimum number of allowed fillers (e.g.  $(\exists_{<3} \text{has\_child})$ ,  $(\exists_{\geq 1} \text{has\_father})$ ). Roles with an implicit ‘ $\exists_{\leq 1}$ ’ number restriction are called *attributes*. These concept specifications are only a subset of all possible specifications. Section 2.2.3 lists the model-theoretic semantics of DL elements mentioned in this paper. The semantics defines the reasoning services a DL inference engine has to provide. In most description logics, the terminology must not contain cyclic definitions because the semantics of cycles cause tremendous theoretical and practical difficulties. Furthermore, a concept name must occur only once on the left-hand side in the definitions of a terminology. The expressiveness of a DL and the tractability of reasoning algorithms for a particular DL depends on the type and possible combinations of connectives and restrictions (see e.g. [21]).

DL systems (i.e. implementations of a DL) usually distinguish two separate reasoning components. The *terminological reasoner* or *classifier* classifies concepts with respect to subsumption relationships between these concepts and organizes them into a taxonomy. The TBox language is designed to facilitate the construction of concept expressions describing classes (types) of individuals. The classifier automatically performs consistency checking of concept definitions and offers retrieval facilities about the classification hierarchy. The (forward-chaining) *assertional reasoner* or *realizer* recognizes and maintains the “type” (i.e. concept membership) of individuals. The purpose of the ABox language is to state constraints or facts (usually restricted to unary or binary predications) that apply to a particular domain or world. Assertional reasoners support a query language in order to access stated and deduced constraints. Some query languages offer the expressiveness of the full first-order calculus.

### 2.2.2 DL: The Concrete Domain

Baader and Hanschke [2, 11] have explored the idea of separating the domain of a description logic into an *abstract* and *concrete* part. An important objective of our approach is to develop a DL formalization of space with two separate domains: the *abstract* and *space domain*. The concept specifications for the abstract domain are used to represent terminological knowledge about spatial objects (e.g. in geography) at an abstract logical level. The (concrete) space domain extends the abstract domain by adding structured mathematical entities for polygons and allows access to efficient reasoning algorithms for concrete spatial regions (e.g. polygons in maps).

A DL with a concrete domain extends a standard DL by adding predicates and individuals for the concrete domain. The predicates can be used to define new concepts in the abstract domain. Role and attribute fillers can be restricted by predicates of the concrete domain.

Basically, (i) the set of predicate names defined by a concrete domain has to be closed under negation and has to contain a predicate name for domain membership, and (ii) the satisfiability problem for finite conjunctions of corresponding predicates has to be decidable (see [11] for a detailed definition).

### 2.2.3 Semantics of DL Elements

Let  $\mathcal{C}$  be the set of concepts,  $\mathcal{R}$  the set of roles, and  $\mathcal{P}$  the set of concrete predicates in a DL theory. The model-theoretic semantics of a DL is based on the notion of an *interpretation* which is defined as a pair  $\langle \mathcal{D}, \xi \rangle$  where  $\mathcal{D} = \mathcal{D}_{\mathcal{C}} \cup \mathcal{D}_{\mathcal{P}}$ ,  $\mathcal{D}_{\mathcal{C}} \cap \mathcal{D}_{\mathcal{P}} = \emptyset$  and  $\xi$  is an assignment function such that  $\xi : \mathcal{C} \rightarrow 2^{(\mathcal{D}_{\mathcal{C}} \cup \mathcal{D}_{\mathcal{P}})}$ ,  $\xi : \mathcal{R} \rightarrow 2^{\mathcal{R}'}$  where  $\mathcal{R}' = (\mathcal{D}_{\mathcal{C}} \times (\mathcal{D}_{\mathcal{C}} \cup \mathcal{D}_{\mathcal{P}}))$ .  $\xi$  must

satisfy the following conditions for mapping syntactical terms to semantical entities (concept names are denoted by  $c$ , role names by  $r$ , and concrete predicate names by  $p$ ). We only list semantics for DL elements mentioned in this paper.

$$\begin{aligned} \xi[(\top)] &= \mathcal{D}_{\mathcal{C}} \\ \xi[(\perp)] &= \emptyset \\ \xi[\text{concept name}] &\subseteq \mathcal{D}_{\mathcal{C}} \text{ or} \\ \xi[\text{concept name}] &\subseteq \mathcal{D}_{\mathcal{P}} \\ \xi[\text{role name}] &\subseteq \mathcal{D}_{\mathcal{C}} \times \mathcal{D}_{\mathcal{C}} \text{ or} \\ \xi[\text{role name}] &\subseteq \mathcal{D}_{\mathcal{C}} \times \mathcal{D}_{\mathcal{P}} \\ \xi[\text{predicate name}] &\subseteq \mathcal{D}_{\mathcal{P}} \\ \xi[(c_1 \wedge \dots \wedge c_n)] &= \bigcap_{i=1}^n \xi[c_i] \\ \xi[(\exists_{\geq n} r)] &= \{x \mid |\{(x, y) \mid (x, y) \in \xi[r]\}| \geq n\} \\ \xi[(\exists_{\leq n} r)] &= \{x \mid |\{(x, y) \mid (x, y) \in \xi[r]\}| \leq n\} \\ \xi[(\forall r c)] &= \{x \mid \forall y : (x, y) \in \xi[r] \Rightarrow y \in \xi[c]\} \\ \xi[(\forall r p)] &= \{x \mid \forall y : (x, y) \in \xi[r] \Rightarrow y \in \xi[p]\} \end{aligned}$$

In the TBox the two special symbols ‘ $\doteq$ ’ and ‘ $\sqsubseteq$ ’ are used for introducing defined and primitive concepts, respectively. The definitions are mapped onto set-inclusion axioms.

- $\text{Cname} \doteq C$  is mapped onto  $\xi[\text{Cname}] = \xi[C]$
- $\text{Cname} \sqsubseteq C$  is mapped onto  $\xi[\text{Cname}] \subseteq \xi[C]$

The semantics of ABox assertions is defined analogously:

- $\text{lname} : C$  is mapped onto  $\text{lname} \in \xi[C]$
- $\langle \text{lname}_1, \text{lname}_2 \rangle : \text{Rname}$  is mapped onto  $(\text{lname}_1, \text{lname}_2) \in \xi[\text{Rname}]$

An interpretation that satisfies all axioms in a terminology is called a *model*. The notion of a model is used to define the reasoning services a DL inference engine has to provide: subsumption and consistency checking which are closely related. A term  $A$  *subsumes* another term  $B$  if and only if for every model  $\langle \mathcal{D}, \xi \rangle$   $\xi[B] \subseteq \xi[A]$  holds. A term  $A$  is *coherent* if and only if there exists a model  $\langle \mathcal{D}, \xi \rangle$  such that  $\xi[A] \neq \emptyset$ .

## 3 A Space Box for Polygons

This section introduces a space box reasoner that realizes inference services over 2D polygons. We demonstrate that the reasoning services provided by current description logics with a concrete domain extension are insufficient for the formalization of space and propose several extensions.

The fundamental idea of the SBox reasoner is the treatment of spatial regions as subsets of  $\mathbb{R}^2$  and to define subsumption between polygons with respect to the relation *g.contains* as defined in Section 2.1 (see also Figure 2). The relation *g.contains* has the properties of an order relation (reflexive, antisymmetric, transitive), i.e. it has the same properties as the subsumption relation for concepts. With this definition

of spatial subsumption we can reduce the satisfiability problem to the decision whether a set of polygons is connected (i.e. there exists a non-empty intersection) or disjoint.

We restrict the concrete predicates to the description of polygons. With the polygon restriction we gain applicability of efficient algorithms (e.g. the simplex procedure) for solving the satisfiability problem. We use concrete predicates for expressing equality ( $\text{equal}_p$ ) or containment ( $\text{g\_inside}_p$ ) of a polygon with respect to the reference polygon  $p$  which is used as the second argument of the relation. We assume an attribute  $\text{has\_area}$  whose filler is from the spatial domain (i.e. a concrete predicate).

For instance, we can now define a concept `northern_german_region` by using the ‘for-all’ constructor ( $\forall r P$ ):

$$\text{northern\_german\_region} \doteq (\forall \text{has\_area } \text{g\_inside}_{p_5})$$

For `northern_german_region` the possible filler of  $\text{has\_area}$  is restricted to a polygon inside of  $p_5$ . The polygon  $p_5$  defines the area of Northern Germany. The construct  $\text{g\_inside}_{p_5}$  subsumes every region of Northern Germany whose associated polygon is  $\text{g\_inside}$  of  $p_5$ . Additionally, we need a concrete predicate for expressing equality of polygons since subsumption of arbitrary subregions is not always desired. For instance, the concept `federal_state_hh` (HH is part of the car license number for Hamburg) contains the equality condition in order to prevent subsumption with subregions of the city of Hamburg area (see also Figure 5):

$$\text{federal\_state\_hh} \doteq \dots \wedge (\forall \text{has\_area } \text{equal}_{p_2})$$

The polygon  $p_2$  defines the area of the federal state Hamburg.  $\text{equal}_{p_2}$  does not subsume any subregion of  $p_2$ . Note that due to the definition of  $\text{g\_inside}$ ,  $\text{g\_inside}_{p_2}$  subsumes  $\text{equal}_{p_2}$ .

We also define similar concrete predicates with respect to the other spatial relations mentioned in the previous section. The idea is to convert a spatial relation into a concept (a one-place predicate) by providing the second argument of the relation as a constant. The reference polygon is required due to the syntax of the concept term constructor. The set of predicates is closed under negation and fulfills the decidability requirement of concrete domains (cf. the notion of ‘admissibility’ of concrete domains).

However, we need more expressivity in order to adequately characterize spatial relations between certain individuals. For instance, the concept definition for `hh_border_district` specifies that every subsumed individual is associated with a polygon that is  $\text{t\_inside}$  of another polygon that, in turn, is referred to by an object which is subsumed by the concept `federal_state_hh`.

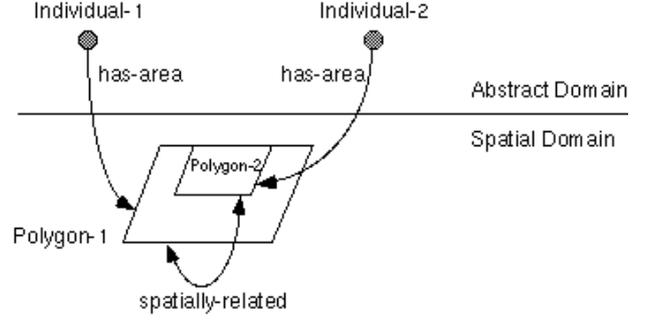


Figure 4: Relationship between abstract and spatial domain

This situation is illustrated in Figure 4.

$$\text{hh\_border\_district} \doteq \dots \wedge (\bigcirc \text{t\_inside } \text{federal\_state\_hh})$$

In order to support this constructor we have to extend existing description logics beyond concrete domains (i.e. we have to extend the description logics defined by Hanschke [11]). In a first approach, we propose a new concept-forming operator  $\bigcirc$  that captures the intuition indicated above. The assignment function is extended for  $\bigcirc$  concept terms (be  $\text{sr}$  a name for an elementary spatial relation as defined in Section 2.1 and  $c$  be a concept term):

$$\begin{aligned} \xi[(\bigcirc \text{sr } c)] = \{x \mid \exists y_1, y_2, z : & (x, y_1) \in \xi[\text{has\_area}], \\ & (z, y_2) \in \xi[\text{has\_area}], \\ & (y_1, y_2) \in \xi[\text{sr}], x \neq z, \\ & z \in \xi[c]\} \end{aligned}$$

Please note that this concept-forming operator is restricted since only elementary spatial relations and not abstract roles are allowed in place of  $\text{sr}$ . We are currently investigating the consequences of this new operator with respect to the completeness of a DL inference algorithm.

Another possible solution to the problems discussed above is the proposal of a role-forming operator that combines abstract attributes and concrete predicates. For instance, by using a *defined* abstract role  $\text{t\_inside\_role}$  we could express the above example as follows:

$$\begin{aligned} \text{t\_inside\_role} & \doteq \exists(\text{has\_area})(\text{has\_area}).\text{t\_inside} \\ \text{hh\_border\_district} & \doteq \dots \wedge (\forall \text{t\_inside\_role } \text{federal\_state\_hh}) \end{aligned}$$

The operator’s formal properties and its applicability are currently under investigation (see also Section 6). The semantics of this new construct is defined as

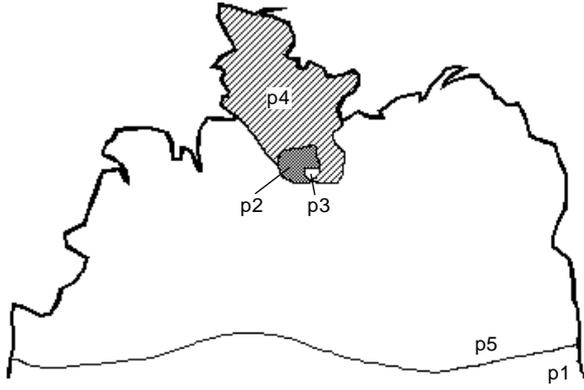


Figure 5: A sketch of the northern part of Germany with polygons for Germany ( $p_1$ ), Northern Germany ( $p_5$ ), the federal states Schleswig-Holstein ( $p_4$ ) and Hamburg ( $p_2$ ) as well as a small district of Hamburg ( $p_3$ ). Polygon  $p_3$  is assumed to be inside  $p_2$  but  $p_2$  is not inside  $p_4$ .

follows (be  $P$  a concrete predicate and  $r_1, r_2$  attributes with concrete individuals as fillers):

$$\xi[\exists(r_1)(r_2).P] = \{(x, y) \mid \exists z_1, z_2 : (x, z_1) \in \xi[r_1], (y, z_2) \in \xi[r_2], (z_1, z_2) \in \xi[P]\}$$

#### 4 Spatioterminological Inferences: an Extended Example

The use of the constructs presented in the previous section is demonstrated with the map example from Figure 1. The surrounding area is presented in Figure 5. Hamburg (represented by polygon  $p_2$ ) is located in Germany ( $p_1$ ), especially in Northern Germany ( $p_5$ ) and directly near the federal state Schleswig-Holstein ( $p_4$ ). The district Öjendorf ( $p_3$ ) of the map in Figure 1 is inside Hamburg. Actually, it is a border district to Schleswig-Holstein.

The formal model is presented with a description logic TBox which is presented below. The areas (polygons) for Germany and Hamburg are explicitly represented with named concepts. The Tbox classifier will detect the implicit subsumption relationship between `german_area` and `hamburg_area`. The concept `german_federal_state` is a primitive concept, i.e. it is defined only with necessary conditions. In our example domain, the filler of `has_area` must be a concrete object which is restricted to `german_area` which, in turn, describes a polygon inside  $p_1$ .

Because the concept `german_federal_state` is primitive, it does not subsume `northern_german_region`. However, due to space subsumption, the concept `northern_german_region` subsumes `federal_state_hh` and also `federal_state_sh` (the concept for Schleswig-Holstein). The concrete predicates  $\min_n$  which means  $[n \dots \infty]$  and  $\max_n$  which means  $[-\infty \dots n]$  are defined

over natural numbers and are also provided by most DL systems.

Another `northern_german_region` is `district_of_hh`. Note again that the area of `district_of_hh` is not a `german_federal_state` because this concept is primitive (see above). The same holds for `district_of_sh`.

`german_area`  $\doteq$  `g_insidep1`

`hamburg_area`  $\doteq$  `g_insidep2`

`german_federal_state`  $\sqsubseteq$   
( $\forall$  `has_area` `german_area`)

`northern_german_region`  $\doteq$   
( $\forall$  `has_area` `g_insidep5`)

`federal_state_hh`  $\doteq$   
`german_federal_state`  $\wedge$  ( $\forall$  `has_area` `equalp2`)

`federal_state_sh`  $\doteq$   
`german_federal_state`  $\wedge$  ( $\forall$  `has_area` `equalp4`)

`district_of_hh`  $\doteq$   
( $\forall$  `has_area` `hamburg_area`)

`district_of_sh`  $\doteq$   
( $\forall$  `has_area` `g_insidep4`)

`sh_border_district`  $\doteq$   
( $\forall$  `has_area` `g_insidep4`)  $\wedge$   
( $\bigcirc$  `t_inside` `federal_state_sh`)

`hh_border_district`  $\doteq$   
`district_of_hh`  $\wedge$  ( $\bigcirc$  `t_inside` `federal_state_hh`)

`hh_border_district_to_sh`  $\doteq$   
`district_of_hh`  $\wedge$   
( $\bigcirc$  `spatially_related` `federal_state_hh`)  $\wedge$   
( $\bigcirc$  `touching` `federal_state_sh`)

While the implicit subsumption relationships discussed above are quite obvious, the last two concept definitions provide more difficult examples. Based on the definitions given above, it can be proven that `hh_border_district_to_sh` is subsumed by `hh_border_district`. A `hh_border_district` is a `district_of_hh` which touches the area of `federal_state_hh` from the inside (relation `t_inside`). The polygon of `federal_state_hh` is explicitly given by the predicate `equalp2` (see the concept definition of `federal_state_hh`). If a `district_of_hh` touches the polygon of `federal_state_sh`, its corresponding area must be tangentially inside the polygon of `federal_state_hh`.

A TBox classifier that deals with the semantics of spatial relations must find these implicit subsumption relationships in order to correctly and completely classify the terminological knowledge base.

In most DL systems, a set of rules can be defined to assert additional constraints for ABox instances when certain conditions (represented by a concept term) are met. For instance, in Hamburg and Schleswig-Holstein, the mountain height is less than 1000 (me-

ters). This relationship is asserted by a rule (operator  $\rightarrow$ ) that fires for every individual that is classified as a member of the concept `northern_german_region`. The second rule adds additional constraints to instances of `district_of_hh`.

```
northern_german_region  $\rightarrow$ 
  ( $\forall$  mountain_height max1000)
district_of_hh  $\rightarrow$ 
  ( $\forall$  zip_code min20000)  $\wedge$ 
  ( $\forall$  area_descriptor min1000)
```

Automatic classification is also important for assertional knowledge defined in the ABox. The following statements define partial information about individuals in our example domain.

```
hamburg : federal_state_hh
öjendorf : district_of_hh
(öjendorf, p3) : has_area
vierlande : ( $\forall$  has_area g_insidep4)  $\wedge$ 
  ( $\bigcirc$  touching ( $\forall$  has_area equalp3))  $\wedge$ 
  ( $\bigcirc$  spatially_related federal_state_sh))
```

The individual `hamburg` is declared to be an instance of `federal_state_hh`. The individual `öjendorf` is a `district_of_hh`. The filler of the `has_area` role for `öjendorf` is `p3`. The ABox reasoner computes that the federal state `hamburg` and the district `öjendorf` are each subsumed by `northern_german_region`, i.e. the mountain heights in the associated areas are less than 1000 meters (see the rules defined above). This kind of derived information can be used to guide the map interpretation process by applying conceptual background knowledge. If the number 7434 in Figure 1 were asserted as a filler for the mountain height of `öjendorf`, the ABox would derive an inconsistency which indicates that another hypothesis has to be tried.

In the last assertion, another individual (named `vierlande`) which touches the polygon of `öjendorf` is defined. Since `vierlande` is by definition subsumed by ( $\forall$  has\_area g\_inside<sub>p<sub>4</sub></sub>), it cannot be a `district_of_hh` but must be inside of `district_of_sh`. However, it touches the `öjendorf` polygon (`p3`) and therefore, it must be automatically classified as a `sh_border_district`.

The examples illustrate the importance of complete inference algorithms for TBox, ABox and SBox classification. For instance, if the implicit subsumption relationship between `hh_border_district` and `hh_border_district_to_sh` were not detected, we could declare an instance of `hh_border_district_to_sh` and claim that a valid zip code in this area might be 7434 which is certainly inconsistent (cf. the rule definition

for `district_of_hh`). Another hypothesis is that 7434 might be an area descriptor. This hypothesis is consistent with the terminological background knowledge defined in our TBox example and might be used as an intermediate result for further interpretation steps.

## 5 Related Work

The idea of incorporating conceptual knowledge (especially knowledge that can be modeled with a decidable description logic) into spatial reasoning and image interpretation problems has been proposed in [10]. Rather than Reiter and Mackworth (see the description of MAPSEE in [17]), who use first order predicate logic, we use a description logic as a basis for our image interpretation problems. In order to be able to validate the image interpretation knowledge itself (i.e. the TBox), we cannot include a domain closure axiom, i.e. we cannot enumerate all objects in every image to be interpreted. In other words: Neither can the problem be reduced to model checking nor to satisfiability checking in propositional logic. Lange and Schröder [12] also discuss the problem of image interpretation in a formal, logical framework. The incorporation of concrete domain predicates for image interpretation problems is presented by Schröder and Neumann [19].

Many other approaches for modeling spatial objects and their relations have been published. The ontological assumptions for the approach presented in this paper are based on a Newtonian conception of space (see [4]). In contrast to the Leibnizian conception (assuming space to be strictly dependent on the relations holding between physical objects), the cartesian structure of our concrete domain approach allows spatial relations to be defined by topological relations between areas defined by polygons (with an external or absolute reference system). The Leibnizian conception has been adopted in many approaches inspired by natural language interpretation problems. Due to space limitations, we cannot discuss the large amount of work on logical models of space in this area.

Grigni et al. [8] study the computational problems in developing an inference system for checking the satisfiability of (conjunctive) combinations of spatial relations. This work is important for us for checking the consistency of combinations of concept terms containing predicates based on spatial relations. Grigni et al. point out that in topological inferences the aspects of relational consistency and planarity interact in rather complex ways. They showed that besides the relational consistency problem a planarity problem has to be solved when areas are assumed to be disjoint. With this additional restriction, in many cases the complexity of the satisfiability problem becomes NP-hard. Lemon [13] showed that in some “spatial” logics based on convex regions one can construct consistent sentences that have no models in the intended geo-

metrical interpretation, i.e. the logics are incomplete with respect to the intended geometrical interpretation (e.g. this has been proven for RCC introduced in [16]).

## 6 Conclusion and Ongoing Work

In this paper, we have demonstrated that topological relations directly influence the kind of conceptual or terminological knowledge that can (and must) be derived by a formal inference engine. On the other hand, assertions about concepts restrict the set of possible spatial relations between different individuals.

We have seen that the use of incomplete reasoning services in practical applications is problematic. For instance, in our application domain the reasoning service might be used to test whether the hypothesis “7434 is the zip code of Öjendorf” is consistent. In the example above we have seen that the correct answer depends on complete TBox classification algorithms. In this case, an incomplete reasoner that does not detect the implicit subsumption relationships in the TBox (see the discussion above) must answer “may be”. However, if we pose the negated query “Is 7434 definitely not the zip code of Öjendorf” the answer must also be “may be” because an inconsistency cannot be derived due to incompleteness. The question is whether “may be” answers can be used for solving problems in a geographical information system, especially when “may be” happens to be interpreted as “no.” Similar problems are likely to occur in incomplete approaches (see e.g. [18] for an image interpretation approach that uses an undecidable description logic).

One idea of the approach presented in this paper is to reduce the complexity of the reasoning algorithms by also considering quantitative spatial data which are available in many practical applications. If concrete polygons are given, no relational consistency checking (see above) is required but standard algorithms from computational geometry can be used. In our map interpretation scenario, the incorporation of a space box with a Newtonian view (i.e. with quantitative data) helps to avoid problems of so-called “spatial” logics. We have discussed some arguments that dealing only with qualitative relations like the ones used by Egenhofer neglects some aspect of space ([8], [13]) when, for instance, the qualitative calculus implies additional properties of geometric objects such as convexity or disjointness of regions.

The SBox extension proposed in this paper is no general geometrical theorem prover. The advantage of our approach which is based also on quantitative information about spatial regions is that the satisfiability test for finite conjunctions of predicates can be reduced to well-known algorithms in computational geometry (basically polygon intersection). Qualita-

tive relations that are grounded in quantitative data provide a bridge to conceptual knowledge and allow more extensive reasoning services to be exploited for solving practical problems.

The treatment of predicate concept terms such as  $(\forall \text{ has\_area } g\_inside_p)$  and  $(\forall \text{ has\_area } equal_p)$  is supported by the work of Hanschke on concrete domain extensions. A prototype implementation using the CLASSIC [5] description logic (and its extension interface) demonstrates that the concept constructor  $\bigcirc$  can be integrated into CLASSIC. However, it demonstrates also the disadvantages of this concept term because it cannot be freely combined with other language constructs. Therefore, we are currently investigating the formal properties of the role operator  $\exists(r_1)(r_2).P$  which can be more flexibly used.

As Borgo et al. emphasize [4], for spatial reasoning we have to consider mereological aspects as well (e.g. part-whole relations). There are many proposals for integrating part-whole relations into description logics and we are investigating ways to combine these approaches with the spatial domain. The examples presented in this paper show that interesting application problems can be solved with an extended description logic supporting reasoning services about qualitative relations.

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## References

- [1] L.C. Aiello, J. Doyle, and S. Shapiro, editors. *Fifth International Conference on Principles of Knowledge Representation, Cambridge, Mass., Nov. 5-8, 1996*, November 1996.
- [2] F. Baader and P. Hanschke. A Scheme for Integrating Concrete Domains into Concept Languages. In *Twelfth International Conference on Artificial Intelligence, Darling Harbour, Sydney, Australia, Aug. 24-30, 1991*, pages 452–457, August 1991.
- [3] A. Borgida. Description Logics in Data Management. *IEEE Transactions on Knowledge and Data Engineering*, 7(5):671–682, 1995.
- [4] S. Borgo, N. Guarino, and C. Masolo. A Pointless Theory of Space Based On Strong Connection

- and Congruence. In Aiello et al. [1], pages 220–229.
- [5] R.J. Brachman, D.L. McGuinness, P.F. Patel-Schneider, L.A. Reswnick, and A. Borgida. Living with Classic: When and How to Use a KL-ONE-like Language. In J.F. Sowa, editor, *Principles of Semantic Networks: Explorations in the Representation of Knowledge*, pages 401–456, San Mateo, California, 1991. Morgan Kaufmann Publishers.
- [6] E. Clementini, P. Di Felice, and P. van Oosterom. A Small Set of Formal Topological Relationships Suitable for End-User Interaction. In D. Abel and B.C. Ooi, editors, *Advances in Spatial Databases, Third International Symposium, SSD'93, Singapore, June 23-25, 1993*, volume 692 of *Lecture Notes in Computer Science*, pages 277–295. Springer Verlag, Berlin, June 1993.
- [7] M.J. Egenhofer. Reasoning about Binary Topological Relations. In O. Günther and H.-J. Schek, editors, *Advances in Spatial Databases, Second Symposium, SSD'91, Zurich, Aug. 28-30, 1991*, volume 525 of *Lecture Notes in Computer Science*, pages 143–160. Springer Verlag, Berlin, August 1991.
- [8] M. Grigni, D. Papadias, and C. Papadimitriou. Topological Inference. In C. Mellish, editor, *Fourteenth International Joint Conference on Artificial Intelligence, Montreal, Quebec, Canada, Aug. 20-25, 1995*, pages 901–906, August 1995.
- [9] V. Haarslev. Formal Semantics of Visual Languages using Spatial Reasoning. In *1995 IEEE Symposium on Visual Languages, Darmstadt, Germany, Sep. 5-9*, pages 156–163. IEEE Computer Society Press, September 1995.
- [10] V. Haarslev, R. Möller, and C. Schröder. Combining Spatial and Terminological Reasoning. In B. Nebel and L. Dreschler-Fischer, editors, *KI-94: Advances in Artificial Intelligence – Proc. 18th German Annual Conference on Artificial Intelligence, Saarbrücken, Sept. 18–23, 1994*, volume 861 of *Lecture Notes in Artificial Intelligence*, pages 142–153. Springer Verlag, Berlin, September 1994.
- [11] P. Hanschke. *A Declarative Integration of Terminological, Constraint-based, Data-driven, and Goal-directed Reasoning*. Infix, Sankt Augustin, 1996.
- [12] H. Lange and C. Schröder. Analysis and Interpretation of Changes in Aerial Images: Knowledge Interpretation and Spatial Reasoning. In H. Ebner, C. Heipke, and K. Eder, editors, *ISPRS Commission III Symposium – Spatial Information from Digital Photogrammetry and Computer Vision, Munich, Germany, Sep. 5–9, 1994*, volume 30 of *International Archives of Photogrammetry and Remote sensing*, pages 475–482, September 1994.
- [13] O. Lemon. Semantical Foundations of Spatial Logics. In Aiello et al. [1], pages 212–219.
- [14] D.L. McGuinness, L.A. Reswnick, and C. Isbell. Description Logic in Practice: A CLASSIC Application. In *IJCAI'95, 14th International Conference on Artificial Intelligence*, pages 2045–2046. Morgan-Kaufmann Publ., 1995.
- [15] B. Nebel. *Reasoning and Revision in Hybrid Representation Systems*, volume 422 of *Lecture Notes in Artificial Intelligence*. Springer Verlag, Berlin, 1990.
- [16] D.A. Randell, Z. Cui, and A.G. Cohn. A Spatial Logic based on Regions and Connections. In B. Nebel, C. Rich, and W. Swartout, editors, *Principles of Knowledge Representation and Reasoning, Cambridge, Mass., Oct. 25-29, 1992*, pages 165–176, October 1992.
- [17] R. Reiter and A.K. Mackworth. A Logical Framework for Depiction and Image Interpretation. *Artificial Intelligence*, 41:125–155, 1989.
- [18] T.A. Russ, R.M. MacGregor, B. Salemi, K. Price, and R. Nevatia. VEIL: Combining Semantic Knowledge with Image Understanding. In *ARPA Image Understanding Workshop*, 1996.
- [19] C. Schröder and B. Neumann. On the Logics of Image Interpretation: Model-Construction in a Formal Knowledge-Representation Framework. In *Proceedings of the 1996 IEEE International Conference on Image Processing ICIP-96, Lausanne, September 16-19, 1996*, volume 2, pages 785–788. IEEE Computer Society Press, September 1996.
- [20] E. Spanier. *Algebraic Topology*. McGraw-Hill Book Company, New York, N.Y., 1966.
- [21] W.A. Woods and J.G. Schmolze. The KL-ONE Family. In F. Lehmann, editor, *Semantic Networks in Artificial Intelligence*, pages 133–177. Pergamon Press, Oxford, England, 1992.
- [22] J.R. Wright, E.S. Weixelbaum, G.T. Vesonder, K. Brown, S.R. Palmer, J.I. Berman, and H.H. Moore. A Knowledge-Based Configurator That Supports Sales, Engineering, and Manufacturing at AT&T Network Systems. *AI Magazine*, 14(3):69–80, 1993.