Combining cardinal direction relations and other orientation relations in QSR*†

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Abstract

We propose a calculus, $c\mathcal{COA}$, combining, thus more expressive than each of, two orientation calculi well-known in QSR: Frank's projection-based cardinal direction calculus, $\mathcal{CDA}$, and a coarser version, $\mathcal{ROA}$, of Freksa's relative orientation calculus. An original constraint propagation procedure, *PcS4c+()*, for $c\mathcal{COA}$-CSPs is presented, which aims at (1) achieving path consistency (*Pc*) for the $\mathcal{CDA}$ projection; (2) achieving strong 4-consistency (*S4c*) for the $\mathcal{ROA}$ projection; and (3) more (+) —the "+" consists of the implementation of the interaction between the two combined calculi. Dealing with the first two points is not new, and involves mainly the $\mathcal{CDA}$ composition table and the $\mathcal{ROA}$ composition table, which can be found in, or derived from, the literature. The originality of the propagation algorithm comes from the last point. Two tables, one for each of the two directions $\mathcal{CDA}$-to-$\mathcal{ROA}$ and $\mathcal{ROA}$-to-$\mathcal{CDA}$, capturing the interaction between the two kinds of knowledge, are defined, and used by the algorithm. The importance of taking into account the interaction is shown with a real example providing an inconsistent knowledge base, whose inconsistency (a) cannot be detected by reasoning separately about each of the two components of the knowledge, just because, taken separately, each is consistent, but (b) is detected by the proposed algorithm, thanks to the interaction knowledge propagated from each of the two compnents to the other.

Key words: Qualitative spatial reasoning, Cardinal directions, Relative orientation, Constraint satisfaction, Path consistency, Strong 4-consistency.

*Qualitative Spatial Reasoning.

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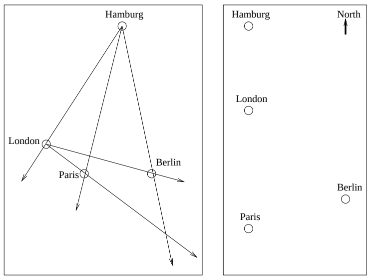

Figure 1: A model for the $\mathcal{ROA}$ component (left), and a model for the $\mathcal{CDA}$ component (right), of the knowledge in Example 1.

1 Introduction

Two important, and widely known calculi for the representation and processing of qualitative orientation are the calculus of cardinal directions, $\mathcal{CDA}$, developed by Frank [5], and the relative orientation calculus developed by Freksa [6]. The former uses a global, south-north/west-east reference frame, and represents knowledge as binary relations on (pairs of) 2D points. The latter allows for the representation of relative knowledge as ternary relations on (triples of) 2D points. Both kinds of knowledge are of particular importance, especially in GIS (Geographic Information Systems) and in robot navigation.

The aim of this work is to look at the importance of combining the two orientation calculi mentioned above. Considered separately, Frank's calculus [5] represents knowledge such as "Hamburg is north-west of Berlin", whereas Freksa's relative orientation calculus [6] represents knowledge such as "You see the main train station on your left when you walk down to the cinema from the university". We propose a calculus, $c\mathcal{COA}$, combining $\mathcal{CDA}$ and a coarser version, $\mathcal{ROA}$, of Freksa's calculus. $c\mathcal{COA}$ allows for more expressiveness than each of the combined calculi, and represents, within the same base, knowledge such as the one in the following example.

Example 1 *Consider the following knowledge on four cities, Berlin, Hamburg, London and Paris: (1) viewed from Hamburg, Berlin is to the left of Paris, Paris is to the left of London, and Berlin is to the left of London; (2) viewed from London, Berlin is to the left of Paris; (3) Hamburg is to the north of Paris, and north-west of Berlin; and (4) Paris is to the south of London. The first two sentences express the* $\mathcal{ROA}$ *component of the knowledge (relative orientation relations on triples of the four cities), whereas the other two express the* $\mathcal{CDA}$ *component of the knowledge (cardinal direction relations on pairs of the four cities).*[1] *Considered separately, each of the two components is consistent, in the sense that one can find an assignment*

[1]Two cardinal direction calculi, to be explained later, are known from Frank's work [5]; we assume in this example the one in Figure 2(right).

of physical locations to the cities that satisfies all the constraints of the component —see the illustration in Figure 1. However, considered globally, the knowledge is clearly inconsistent (the physical locations assigned to Hamburg, London and Paris form a triangle in any model of the $\mathcal{ROA}$ component, whereas they are collinear in any model of the $\mathcal{CDA}$ component).

Example 1 clearly shows that reasoning about combined knowledge consisting of an $\mathcal{ROA}$ component and a $\mathcal{CDA}$ component, e.g., checking its consistency, does not reduce to a matter of reasoning about each component separately —reasoning separately about each component in the case of Example 1 shows two components that are both consistent, whereas the conjunction of the knowledge in the two components is inconsistent. As a consequence, the interaction between the two kinds of knowledge has to be handled. With this in mind, a constraint propagation procedure, *PcS4c+()*, for $c\mathcal{COA}$-CSPs is proposed, which aims at: (1) achieving path consistency (*Pc*) for the $\mathcal{CDA}$ projection; (2) achieving strong 4-consistency (*S4c*) for the $\mathcal{ROA}$ projection; and (3) more (+). The procedure does more than just achieving path consistency for the $\mathcal{CDA}$ projection, and strong 4-consistency for the $\mathcal{ROA}$ projection. It implements as well the interaction between the two combined calculi. The procedure is, to the best of our knowledge, original.

In the remainder of the paper, we first give a brief description of the propagation algorithm we propose, including its part dealing with the interaction knowledge, consisting of $\mathcal{ROA}$ knowledge inferred from $\mathcal{CDA}$ knowledge, and, conversely, of $\mathcal{CDA}$ knowledge inferred from $\mathcal{ROA}$ knowledge. We then reconsider our illustrating example to show that, thanks to the interaction knowledge, more inconsistencies are detected than one would get from just applying path consistency to the CDA component and four-consistency to the ROA component. We finish with a discussion relating the work to current research on spatio-temporalising the well-known $\mathcal{ALC}(\mathcal{D})$ family of description logics (DLs) with a concrete domain [2]: the discussion shows that if two (spatial) ontologies operate on the same universe of objects (in this work, the universe of 2D points), while using different languages for their knowledge representation, then integrating the two ontologies needs an inference mechanism for the interaction of the two languages, so that, given knowledge expressed in the integrating ontology, consisting of two components (one for each of the integrated ontologies), each of the two components can infer knowledge from the other.

2 Frank's and Freksa's orientation calculi and their integration

2.1 Frank's calculus.

Frank's models of cardinal directions in 2D [5] are illustrated in Figure 2. They use a partition of the plane into regions determined by lines passing through a reference object, say S. Depending on the region a point P belongs to, we have

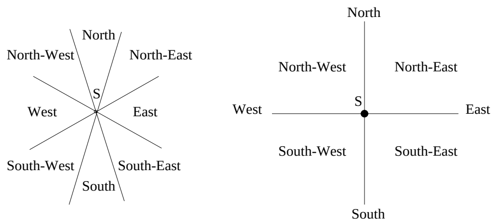

Figure 2: Frank's cone-shaped (left) and projection-based (right) models of cardinal directions.

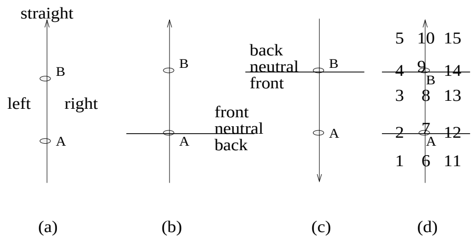

Figure 3: The partition of the universe of 2D positions on which is based Freksa's relative orientation calculus.

$No(P,S)$, $NE(P,S)$, $Ea(P,S)$, $SE(P,S)$, $So(P,S)$, $SW(P,S)$, $We(P,S)$, $NW(P,S)$, or $Eq(P,S)$, corresponding, respectively, to the position of P relative to S being *north*, *north-east*, *east*, *south-east*, *south*, *south-west*, *west*, *north-west*, or *equal*. Each of the two models can thus be seen as a binary Relation Algebra (RA), with nine atoms. Both use a global, *west-east/south-north*, reference frame. We focus our attention on the projection-based model (Figure 2(right)), which has been assessed as being cognitively more adequate [5] (cognitive adequacy of spatial orientation models is discussed in [6]).

2.2 Freksa's calculus.

A well-known model of relative orientation of 2D points is the calculus defined by Freksa [6]. The calculus corresponds to a specific partition, into 15 regions, of the plane, determined by a parent object, say A, and a reference object, say B (Figure 3(d)). The partition is based on the following: (1) the *left/straight/right* partition of the plane determined by an observer placed at the parent object and looking in the direction of the reference object (Figure 3(a)); (2) the *front/neutral/back* partition of the plane determined by the same observer (Figure 3(b)); and (3) the similar *front/neutral/back* partition of the plane obtained when we swap the roles of the parent object and the reference object (Figure 3(c)). Combining the three partitions (a), (b) and (c) of Figure 3 leads to the partition of the universe of 2D positions on which is based the calculus in [6] (Figure 3(d)).

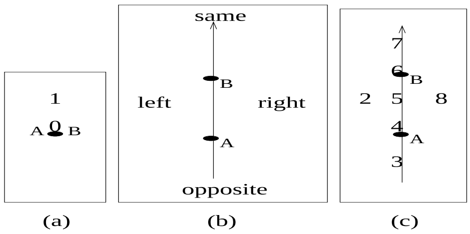

Figure 4: The partition of the universe of 2D positions on which is based the $\mathcal{ROA}$ calculus.

2.3 A new relative orientation calculus.

It is known that, computationally, Freksa's relative orientation calculus, even when restricted to its 15 atoms, behaves badly [15]. We therefore consider a coarser version of it, obtained from the original one by ignoring, in the construction of the partition of the plane determined by a parent object and a reference object (Figure 3(d)), the two *front/neutral/back* partitions (Figure 3(b-c)). In other words, we consider only the *left/straight/right* partition (Figure 3(a)) —we also keep the 5-element partitioning of the line joining the parent object to the reference object. The final situation is depicted in Figure 4, where A and B are the parent object and the reference object, respectively. Figure 4(b-c) depicts the general case, corresponding to A and B being distinct from each other: this general-case partition leads to 7 regions (Figure 4(c)), numbered from 2 to 8, corresponding to 7 of the nine atoms of the calculus, which we refer to as *lr* (to the left of the reference object), *bp* (behind the parent object), *cp* (coincides with the parent object), *bw* (between A and B), *cr* (coincides with the reference object), *br* (behind the reference object), and *rr* (to the right of the reference object). Figure 4(a) illustrates the degenerate case, corresponding to equality of A and B. The two regions, corresponding, respectively, to the primary object coinciding with A and B, and to the primary object distinct from A and B, are numbered 0 and 1. The corresponding atoms of the calculus will be referred to as *de* (degenerate equal) and *dd* (degenerate distinct).

From now on, we refer to the cardinal directions calculus as $\mathcal{CDA}$ (Cardinal Directions Algebra), and to the coarser version of Freksa's relative orientation calculus as $\mathcal{ROA}$ (Relative Orientation Algebra). A $\mathcal{CDA}$ (resp. $\mathcal{ROA}$) relation is any subset of the set of all $\mathcal{CDA}$ (resp. $\mathcal{ROA}$) atoms. A $\mathcal{CDA}$ (resp. $\mathcal{ROA}$) relation is said to be atomic if it contains one single atom (a singleton set); it is said to be the $\mathcal{CDA}$ (resp. $\mathcal{ROA}$) universal relation if it contains all the $\mathcal{CDA}$ (resp. $\mathcal{ROA}$) atoms. When no confusion raises, we may omit the brackets in the representation of an atomic relation.

3 CSPs of cardinal direction relations and relative orientation relations on 2D points

We define a $c\mathcal{COA}$-CSP as a CSP of which the constraints consist of a conjunction of $\mathcal{CDA}$ relations on pairs of the variables, and $\mathcal{ROA}$ relations on triples of the variables. The universe of a $c\mathcal{COA}$-CSP, i.e., the domain of instantiation of its variables, is the continuous set $\mathbb{R}^2$ of 2D points.

3.1 Matrix representation of a $c\mathcal{COA}$-CSP.

A $c\mathcal{COA}$-CSP P can, in an obvious way, be represented as two constraint matrices: a binary constraint matrix, $\mathcal{B}^P$, representing the $\mathcal{CDA}$ part of P, i.e., the subconjunction consisting of $\mathcal{CDA}$ relations on pairs of the variables; and a ternary constraint matrix, $\mathcal{T}^P$, representing the $\mathcal{ROA}$ part of P, i.e., the rest of the conjunction, consisting of $\mathcal{ROA}$ relations on triples of the variables. We refer to the representation as $\langle \mathcal{B}^P, \mathcal{T}^P \rangle$. The $\mathcal{B}^P$ entry $(\mathcal{B}^P)_{ij}$consists of the $\mathcal{CDA}$ relation on the pair (X_i, X_j) of variables. Similarly, the $\mathcal{T}^P$ entry $(\mathcal{T}^P)_{ijk}$ consists of the $\mathcal{ROA}$ relation on the triple (X_i, X_j, X_k) of variables.

3.2 Reasoning within $\mathcal{CDA}$ and the $\mathcal{CDA}$-to-$\mathcal{ROA}$ interaction: the tables.

We present the $\mathcal{CDA}$-to-$\mathcal{ROA}$ interaction in a knowledge base consisting of a $c\mathcal{COA}$-CSP. The other direction, i.e., the $\mathcal{ROA}$-to-$\mathcal{CDA}$ interaction, can be found in the full paper [9]. The table in Figure 5 presents the augmented $\mathcal{CDA}$ composition table; for each pair (r_1, r_2) of $\mathcal{CDA}$ atoms, the table provides: the standard composition, $r_1 \circ r_2$, of r_1 and r_2 [5, 12]; and the most specific $\mathcal{ROA}$ relation $r_1 \otimes r_2$ such that, for all 2D points x, y, z, the conjunction $r_1(x, y) \wedge r_2(y, z)$ logically implies $(r_1 \otimes r_2)(x, y, z)$.

The operation $\circ$ is just the normal composition: it is internal to $\mathcal{CDA}$, in the sense that it takes as input two $\mathcal{CDA}$ atoms, and outputs a $\mathcal{CDA}$ relation. The operation $\otimes$, however, is not internal to $\mathcal{CDA}$, in the sense that it takes as input two $\mathcal{CDA}$ atoms, but outputs an $\mathcal{ROA}$ relation; $\otimes$ captures the interaction between $\mathcal{CDA}$ knowledge and $\mathcal{ROA}$ knowledge, in the direction $\mathcal{CDA}$-to-$\mathcal{ROA}$, by inferring $\mathcal{ROA}$ knowledge from given $\mathcal{CDA}$ knowledge. As an example for the new operation $\otimes$, from $SE(Berlin, London) \wedge No(London, Paris)$, saying that Berlin is south-east of London, and that London is north of Paris, we infer the $\mathcal{ROA}$ relation lr on the triple $(Berlin, London, Paris)$: $lr(Berlin, London, Paris)$, saying that, viewed from Berlin, Paris is to the left of London.

The reader is referred to [5, 12] for the $\mathcal{CDA}$ converse table, providing the converse $r^\smile$ for each $\mathcal{CDA}$ atom r.

$\frac{\circ}{\otimes}$	*No*	*So*	*Ea*	*We*	*NE*	*NW*	*SE*	*SW*
No	*No*	[*So*, *No*]	*NE*	*NW*	*NE*	*NW*	[*SE*, *NE*]	[*SW*, *NW*]
	br	{*bp*, *cp*, *bw*}	*rr*	*lr*	*rr*	*lr*	*rr*	*lr*
So	[*So*, *No*]	*So*	*SE*	*SW*	[*SE*, *NE*]	[*SW*, *NW*]	*SE*	*SW*
	{*bp*, *cp*, *bw*}	*br*	*lr*	*rr*	*lr*	*rr*	*lr*	*rr*
Ea	*NE*	*SE*	*Ea*	[*We*, *Ea*]	*NE*	[*NW*, *NE*]	*SE*	[*SW*, *SE*]
	lr	*rr*	*br*	{*bp*, *cp*, *bw*}	*lr*	*lr*	*rr*	*rr*
We	*NW*	*SW*	[*We*, *Ea*]	*We*	[*NW*, *NE*]	*NW*	[*SW*, *SE*]	*SW*
	rr	*lr*	{*bp*, *cp*, *bw*}	*br*	*rr*	*rr*	*lr*	*lr*
NE	*NE*	[*SE*, *NE*]	*NE*	[*NW*, *NE*]	*NE*	[*NW*, *NE*]	[*SE*, *NE*]	?
	lr	*rr*	*rr*	*lr*	{*lr*, *br*, *rr*}	*lr*	*rr*	{*lr*, *bp*, *cp*, *bw*, *rr*}
NW	*NW*	[*SW*, *NW*]	[*NW*, *NE*]	*NW*	[*NW*, *NE*]	*NW*	?	[*SW*, *NW*]
	rr	*lr*	*rr*	*lr*	*rr*	{*lr*, *br*, *rr*}	{*lr*, *bp*, *cp*, *bw*, *rr*}	*lr*
SE	[*SE*, *NE*]	*SE*	*SE*	[*SW*, *NE*]	[*SE*, *NE*]	?	*SE*	[*SW*, *NE*]
	lr	*rr*	*lr*	*rr*	*lr*	{*lr*, *bp*, *cp*, *bw*, *rr*}	{*lr*, *br*, *rr*}	*rr*
SW	[*SW*, *NW*]	*SW*	[*SW*, *SE*]	*SW*	?	[*SW*, *NW*]	[*SW*, *SE*]	*SW*
	rr	*lr*	*lr*	*rr*	{*lr*, *bp*, *cp*, *bw*, *rr*}	*rr*	*lr*	{*lr*, *br*, *rr*}

Figure 5: The augmented composition table of the cardinal directions calculus: for each pair (r_1, r_2) of $\mathcal{CDA}$ atoms, the table provides the composition, $r_1 \circ r_2$, of r_1 and n r_2, as well as the most specific $\mathcal{ROA}$ relation $r_1 \otimes r_2$ such that, for all 2D points x, y, z, the conjunction $r_1(x, y) \wedge r_2(y, z)$ logically implies $(r_1 \otimes r_2)(x, y, z)$; the question mark symbol ? represents the $\mathcal{CDA}$ universal relation $\{No, NW, We, SW, So, SE, Ea, NE, Eq\}$.

3.3 A constraint propagation procedure for $c\mathcal{COA}$-CSPs.

We propose a constraint propagation procedure, *PcS4c+()*, for $c\mathcal{COA}$-CSPs, which aims at:

1. achieving path consistency (*Pc*) for the $\mathcal{CDA}$ projection, using, for instance, the algorithm in [1];
2. achieving strong 4-consistency (*S4c*) for the $\mathcal{ROA}$ projection, using, for instance, the algorithm in [11]; and
3. more (+).

The procedure does more than just achieving path consistency for the $\mathcal{CDA}$ projection, and strong 4-consistency for the $\mathcal{ROA}$ projection. It implements as well the interaction between the two combined calculi; namely:

1. The path consistency operation, $(\mathcal{B}^P)_{ik} \leftarrow (\mathcal{B}^P)_{ik} \cap (\mathcal{B}^P)_{ij} \circ (\mathcal{B}^P)_{jk}$, which, under normal circumstances, operates internally, within a same CSP, is now augmented so that it can send information from the $\mathcal{CDA}$ component into the $\mathcal{ROA}$ component.
2. The strong 4-consistency operation, $(\mathcal{T}^P)_{ijk} \leftarrow (\mathcal{T}^P)_{ijk} \cap (\mathcal{T}^P)_{ijl} \circ (\mathcal{T}^P)_{ilk}$, which also operates internally under normal circumstances, is augmented so that it can send information from the $\mathcal{ROA}$ component into the $\mathcal{CDA}$ component.

The reader is referred to the full version of the work for details [9].

Example 2 *Consider again the description of Example 1. We can represent the situation as a c$\mathcal{COA}$-CSP with variables X_b, X_h, X_l and X_p, standing for the cities of Berlin, Hamburg, London and Paris, respectively.*

1. *The knowledge "viewed from Hamburg, Berlin is to the left of Paris" translates into the $\mathcal{ROA}$ constraint* $\mathrm{lr}(X_h, X_p, X_b)$*:* $(\mathcal{T}^P)_{hpb} = \{\mathrm{lr}\}$.

2. *The other $\mathcal{ROA}$ knowledge translates as follows:* $(\mathcal{T}^P)_{hlp} = \{\mathrm{lr}\}$, $(\mathcal{T}^P)_{hlb} = \{\mathrm{lr}\}$, $(\mathcal{T}^P)_{lpb} = \{\mathrm{lr}\}$.

3. *The $\mathcal{CDA}$ part of the knowledge translates as follows:* $(\mathcal{B}^P)_{hp} = \{\mathrm{No}\}$, $(\mathcal{B}^P)_{hb} = \{\mathrm{NW}\}$, $(\mathcal{B}^P)_{pl} = \{\mathrm{So}\}$.

As discussed in Example 1, reasoning separately about the two components of the knowledge shows two consistent components, whereas the combined knowledge is clearly inconsistent. Using the procedure PcS4c+()*, we can detect the inconsistency in the following way. From the $\mathcal{CDA}$ constraints* $(\mathcal{B}^P)_{hp} = \{\mathrm{No}\}$ *and* $(\mathcal{B}^P)_{pl} = \{\mathrm{So}\}$*, the algorithm infers, using the augmented $\mathcal{CDA}$ composition table of Figure 5 —specificaly, the $\mathcal{CDA}$-to-$\mathcal{ROA}$ interaction operation $\otimes$— the $\mathcal{ROA}$ relation* $\{\mathrm{bp}, \mathrm{cp}, \mathrm{bw}\}$ *on the triple* (X_h, X_p, X_l)*. The conjunction of the inferred knowledge* $\{\mathrm{bp}, \mathrm{cp}, \mathrm{bw}\}(X_h, X_p, X_l)$ *and the already existing knowledge* $\{\mathrm{lr}\}(X_h, X_l, X_p)$ *—equivalent to* $\{\mathrm{rr}\}(X_h, X_p, X_l)$*— gives the empty relation, indicating the inconsistency of the knowledge.*

4 Discussion

Current research shows clearly the importance of developing spatial RAs: specialising an $\mathcal{ALC}(\mathcal{D})$-like Description Logic (DL) [2], so that the roles are temporal immediate-successor (accessibility) relations, and the concrete domain is generated by a decidable spatial RA in the style of the well-known Region-Connection Calculus RCC-8 [14], leads to a computationally well-behaving family of languages for spatial change in general, and for motion of spatial scenes in particular:

1. Deciding satisfiability of an $\mathcal{ALC}(\mathcal{D})$ concept w.r.t. to a cyclic TBox is, in general, undecidable (see, for instance, [13]).

2. In the case of the spatio-temporalisation, however, if we use what is called weakly cyclic TBoxes in [10], then satisfiability of a concept w.r.t. such a TBox is decidable. The axioms of a weakly cyclic TBox capture the properties of modal temporal operators. The reader is referred to [10] for details.

Spatio-temporal theories such as the ones defined in [10] can be seen as single-ontology spatio-temporal theories, in the sense that the concrete domain represents only one type of spatial knowledge (e.g., RCC-8 relations if the concrete domain is generated by RCC-8). We could extend such theories to handle more than just one concrete domain: for instance, two concrete domains, one generated by $\mathcal{CDA}$, the other by $\mathcal{ROA}$. This would lead to what could be called multi-ontolopgy spatio-temporal theories. The presented work clearly shows that the reasoning issue in such multi-ontology theories does not reduce to reasoning about the projections onto the different concrete domains.

5 Summary

We have presented the combination of two calculi of spatial relations well-known in Qualitative Spatial Reasoning (QSR): Frank's projection-based cardinal direction calculus [4, 5] and Freksa's relative orientation calculus [6, 7]. With an example illustrating the importance of such a combination to Geographical Information Systems (*GIS*), we have shown that reducing the issue of reasoning about knowledge expressed in the combined language to a simple matter of reasoning separately about each of the two components was not sufficient. The interaction between the two kinds of knowledge has thus to be handled: we have provided a constraint propagation algorithm for such a purpose, which:

1. achieves path consistency for the cardinal direction component;
2. achieves strong 4-consistency for the relative orientation component; and
3. implements the interaction between the two kinds of knowledge.

Combining and integrating different kinds of knowledge is an emerging and challenging issue in QSR. Related work has been done by Gerevini and Renz [8], which deals with the combination of topological knowledge and relative size knowledge in QSR. Similar work might be carried out for other aspects of knowledge in QSR, such as qualitative distance [3] and relative orientation [6, 7], a combination known to be highly important for *GIS* and robot navigation applications, and on which not much has been achieved so far.

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