Resistive Networks Revisited: Exploitation of Network Structures and Qualitative Reasoning about Deviations is the Key

Lothar Hotz, Heiko Milde, Ralf Möller, Bernd Neumann
Laboratory for Artificial Intelligence, University of Hamburg
Vogt-Koelln-Str. 30, 22527 Hamburg, Germany
milde@kogs.informatik.uni-hamburg.de

Abstract

For diagnostic purposes, analog circuits may be qualitatively modeled as resistive networks. We demonstrate that approaches to this task show certain weaknesses if they are based on sign-based qualitative values. In order to overcome these deficiencies, we first introduce qualitative deviation values with a semantics that enables us to model different classes of faults arising in analog circuits. The qualitative values adequately describe different effects that faults may have. Then we present a sound and complete inference algorithm for computing these effects using qualitative operators and local propagation techniques.*

1. Introduction

In the past, many different approaches to model-based diagnosis of analog circuits have been published. For instance, if the circuit parameters can be described by crisp quantitative values, a linear network can be analyzed by existing tools such as SPICE (Banzhaf, 1989) or systems based on CLP(R) (e.g. (Biasizzo and Novak, 1995)). In order to cope with tolerances and inaccuracies, the DIANA system (Dague et al., 1990) uses quantitative *intervals* to describe network parameters. The FLAMES system (Mohamed and Marzouki, 1996) proposes *fuzzy* intervals to describe inaccuracies more adequately.

While these systems can be used to simulate a large class of analog circuits by exploiting detailed component models, (Struss et al., 1995) argue that for diagnostic purposes more abstract models are advantageous. In particular, resistive networks with qualitative parameter values have been investigated in the literature.

For instance, adhering to the no-function-in-structure principle, the Connectivity Method (Struss et al., 1995) basically propagates qualitative information that encodes which port of a circuit component is connected to source or sink. However, not all kinds of circuits can be handled adequately. In order to overcome these deficiencies, (Mauss and Neumann, 1996) have developed a qualitative method to analyze resistive networks by exploiting the *structure* of networks. The so-called SPS method explicitly represents a network's series-parallel-star structure as a tree (sps-tree). As a result of the network analysis, for all currents and voltages, *sign-based* qualitative values are determined. In our opinion, the Connectivity Method and the SPS method focus on the detection of *structural faults*, e.g. broken wires or comparable component faults such as blown light bulbs etc. They do not address, however, several other diagnosis topics which may arise in applications:

- non-structural faults such as slight deviations from normal behavior,
- deviative effects of non-structural and structural faults,
- specific circuit topologies,
- dealing with abstractions in diagnosis models,
- dealing with variants.

This paper presents a qualitative method for these topics. The problems solved by the approach are explained with an application example. In particular, we consider a field regulator that is a subcomponent of a motor. A schematic diagram of the field regulator circuit is presented in Figure 1. The components shown in the figure are abstractions of the real physical components. For instance, the control switches T1 to T4 are actually implemented with transistors and diodes but, for diagnostic purposes, such a fine-grained representation is not required.

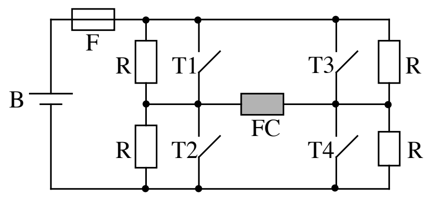

Figure 1: Field regulator (resistors (R), fuse (F), battery (B), field coil (FC), controlled switches (T1 to T4)).

In our application we focus on non-structural faults such as slight deviations from normal behavior, e.g. increased resistance values. Faults of this kind can neither be *modeled* by the SPS method nor by the Connectivity Method because of the sign-based qualitative values used by these methods. In analog circuits the occurrence of a fault affects all currents and voltages, i.e. the absolute values of parameters change. However, in most cases, the parameters do not change in their signs (or reach certain fixed landmarks). Thus, it is hardly possible to adequately derive these fault *effects* using sign-based qualitative values. Furthermore, bridge circuit topologies are relevant in our domain (see Figure 1). These circuits pose a special problem for qualitative approaches because the direction of the current through the bridge resistor usually depends on the exact quantitative values of the component parameters.

Since the components of our model are abstractions of real components, *quantitative* modeling systems (see above) are not appropriate, either. In addition, we also have to deal

* This research has been supported by the Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie (BMBF) under the grant 01 IN 509 D 0, INDIA.

with the "variants' dilemma" (Struss et al., 1995). This means that a certain model of the field regulator should cover several variants of this device. Variants differ only slightly concerning values of component parameters. Thus, in principle, qualitative methods are preferable.

Based on the SPS method we introduce a new qualitative approach for reasoning about analog circuits for diagnostic purposes. The main features of our method are:

- The qualitative values represent deviations as well as sign information. With deviations we can describe non-structural faults such as "resistance too high" as well as structural faults such as blown light bulbs ("resistance too high and infinite") even in bridge circuit topologies.
- The semantics of the qualitative values is grounded in the quantitative nature of landmarks and their algebraic relations (e.g. order relations). This way we can show soundness and completeness of the derivation algorithm for qualitative reasoning, cf. also (Struss, 1990).
- We show that it is not necessary to specify the absolute quantitative values of landmarks. Considering the order relation between landmarks allows us to deal with abstract circuit components and provides a basis for dealing with the "variants' dilemma". The method presented in this paper derives simulation results that are sufficient for fault discrimination in a diagnosis application.

The key idea of our approach is (i) to derive a set of qualitative values (deviation values) to adequately describe faults and their effects and (ii) to define qualitative operators to propagate these values in order to simulate circuit behavior. The paper is structured as follows. Qualitative values for describing deviations are introduced in Section 2. In Section 3 we describe a qualitative calculus simulating circuit behavior based on deviations. Section 4 points out the main achievements in a conclusion.

2. Qualitative Values

In a resistive network, there are currents and voltages whose directions can be determined by the structure of the network. These currents and voltages as well as resistances can be adequately described by a set of qualitative values that cover the extended *positive* real number line $[0, \infty]$ (Type 1). For some currents and voltages, the directions are not determined by the network structure. Thus, we also need qualitative values that cover the *whole* extended real number line $[-\infty, \infty]$ (Type 2). Distinguishing between two types of qualitative value sets is not merely a syntactic criterion but sharpens the reasoning about effects in resistive networks.

Qualitative Values, Type 1. The qualitative values of Type 1 and their semantics are shown in Table 1. Although the landmark values in the semantics definition are not specified as fixed quantitative values, we do rely on the order between landmarks: $0 < Amin < Amax < \infty$. The qualitative value *A_normal* represents an interval that encodes the range of parameter A in the faultless state. The other qualitative values describe deviations from the faultless state. We distinguish between extreme deviation values *A_low_0* and *A_high_inf* and non-extreme deviation values *A_low* and *A_high*.

qualitative value	abbreviation	semantics
A_low_0	A_0	$A \in [0, 0]$
A_low	A_l	$A \in (0, Amin)$
A_normal	A_n	$A \in [Amin, Amax]$
A_high	A_h	$A \in (Amax, \infty)$
A_high_inf	A_∞	$A \in [\infty, \infty]$

Table 1: Qualitative values of Type 1

Describing resistances (R) qualitative values of Type 1 can be used to model structural as well as non-structural faults. On the one hand, structural faults such as short circuits and broken wires can be described by the extreme deviations *R_low_0* and *R_high_inf,* respectively. On the other hand, non-structural faults such as partial short circuits in coils and corroded wiring points can be modeled by the qualitative values *R_low* and *R_high*, respectively. The faultless state of a component is represented by the qualitative value *R_normal* which represents an interval. This enables us to be tolerant with respect to differential deviations such as physical tolerances and temperature drifts.

Voltages (U) and currents (I) can also be described by qualitative values of Type 1 if their directions are determined by the structure of the network. Again, the qualitative value *U/I_normal* characterizes the faultless state. Structural and non-structural faults mostly lead to non-extreme deviations. For instance, there is lower current (*I_low*) through corroded wiring points. Note that in sign-based approaches (e.g. (Mauss and Neumann, 1996)) effects like these cannot be modeled. Structural faults can result in extreme deviation values, e.g. there is no current (*I_low_0*) through a broken wire.

Qualitative Values, Type 2. The qualitative values of Type 2 and their semantics are shown in Table 2.

qualitative value	abbreviation	semantics
A_Low_neg_inf	A_L_∞	$A \in [-\infty, -\infty]$
A_Low	A_L	$A \in (-\infty, Amin)$
A_Normal	A_N	$A \in [Amin, Amax]$
A_High	A_H	$A \in (Amax, \infty)$
A_High_inf	A_H_∞	$A \in [\infty, \infty]$

Table 2: Qualitative values of Type 2

We use capital letters to distinguish the different types. Again, we emphasize the order relation between the landmarks: $-\infty < Amin < Amax < \infty$. Note that, none of the values *U/I_Low*, *U/I_Normal* and *U/I_High* includes sign information.

3. SDSP-Analysis

A resistive network can be described by a system of linear equations based on Kirchhoff's and Ohm's laws. Assuming that a fault has occurred, this system of equations can be exploited in order to determine qualitative values for voltages and currents. That is, the system of equations has to be solved algebraically. Furthermore, signs of partial derivatives of the symbolic solutions of the system of equations have to be determined. Moreover, symbolic expressions describing interval boundaries have to be ordered by size. Although possible, this process seems to be very complicated because the symbolic expressions which are involved can be extremely complex.

One approach to simplify this under specific circumstances has been published by (Mauss and Neumann, 1996). The main advantage of the SPS method is that the network is described by a set of component-oriented local equations which can be solved step by step. The equations are organized in a SP-tree which directly relates corresponding variables. The SPS method currently is a sign-based approach for analyzing resistive networks. In the introduction we have seen that with sign-based approaches not all faults and their different effects can be modeled. Therefore, we adapt the SPS method to the deviation-based qualitative values introduced in Section 2.

3.1 Qualitative Analysis of Resistive Networks

Our approach to the qualitative analysis of a resistive network consists of two main steps.

SDSP Transformation. The circuit is transformed by star-delta transformations and series-parallel reductions to generate an SP-tree. The SP-tree is an explicit representation of the structure of the network that can be used to simulate different kinds of faults.

As a difference to the SPS method we would like to emphasize that we use star as well as delta conversions (hence the name of our method: SDSP method). This is advantageous because, in comparison to the SPS method, new classes of network topologies can be treated. As a further difference, we restrict stars and deltas to be transformed to those with three edges - with the purpose to obtain a fixed number of equation types. This is important because for each equation type a qualitative version has to be defined. As a disadvantage of this restriction to stars and deltas with three edges, we admit that there are some networks that cannot be treated (e.g. networks consisting exclusively of four-edge stars without any delta transformations applicable). According to our experiences, these networks are hardly relevant in practice.

Local propagation of qualitative values. The second step consists of local propagation of qualitative values in the SP-tree in order to simulate circuit behavior (i.e. the step can be carried out for each of the supplied fault models).

First, applying the star-delta transformation, values of transformation resistances are determined.

Second, values of resistances are propagated from the leaves of the SP-tree to its root by exploiting two electrical laws describing compensation resistances of series and parallel groupings.

Third, values for currents and voltages are propagated from the root of the SP-tree to its leaves. For that, four different electrical laws are evaluated, i.e. current divider, voltage divider, same voltages and same currents rule. This step of the network analysis is an extension of the SPS method since we exploit an extended set of electrical laws, i.e. current divider and the voltage divider rules are added. One could argue that these rules violate the no-function-in-structure principle. Nevertheless we do not hesitate to exploit these rules, because using them does not imply any limitations on the applicability of our approach. Furthermore, we show that these two rules are required by the propagation algorithm (see the comments on soundness and completeness in Section 4).

Fourth, values of voltages and currents of the original network are determined by exploiting current and voltage transformations that are a part of the star-delta transformation.

In the transformed network, directions of currents and voltages are determined by the network's structure. Thus, currents and voltages are described by qualitative values of Type 1. Resistances are described by qualitative values of Type 1, too. Therefore, the qualitative versions of equations describing electrical laws mentioned above have to be defined on values of Type 1. In the next section some of these definitions are given.

3.2 Combining Qualitative Values

Our definitions of qualitative versions of electrical laws are based on the following three features. We use uppercase letters to describe qualitative operators, e.g. *SCR* means qualitative *series compensation resistance*.

1. The qualitative values *A_normal* and *A_Normal* represent the faultless state. Therefore, any parameter A has the qualitative value A*_normal (A_Normal)* if its value is determined from parameters that, in turn, have the qualitative values A*_normal (A_Normal)*. E.g., *SCR*(*R1_normal, R2_ normal*) = *S3_normal*. That is, in the faultless state, each parameter is described by *normal*.
2. The qualitative values of the SDSP method have a clear semantics, e.g. *A_normal* <-> $A \in [Amin, Amax]$. We emphasize again that the qualitative values represent symbolic intervals whose boundaries are not quantitatively specified, but they are ordered, e.g. $0 < Amin < Amax < \infty$.
3. Utilizing elementary operators for interval-based evaluations of equations may lead to the selection problem (Struss, 1990), i.e. due to the multiple occurrence of certain variables in equations unnecessary widened intervals may be calculated. In order to overcome this deficiency, for each electrical law that is utilized for local propagation of qualitative values, we explicitly define its interval-based one-step evaluation. This is possible because we utilize a limited number of electrical laws. As an example we show how the interval-based one-step evaluation of the *voltage divider rule* is defined. The quantitative version of this rule is described by $U1 = \frac{R1}{R1 + R2} \cdot U3$. In order to define its interval-based evaluation we determine the smallest interval that contains all possible values of U1 under the condition that $R1 \in [R1left, R1right]$ and $R2 \in [R2left, R2right]$ and $U3 \in [U3left, U3right]$ holds. That is, $U1 \in \left[\frac{R1}{R1 + R2} \cdot U3 \Big|_{min}, \frac{R1}{R1 + R2} \cdot U3 \Big|_{max} \right]$ is determined. Note that $U3, R1, R2 \geq 0$ because the voltage divider rule is only applied when SP-reducible net-

works are considered. In this case, currents and voltages have non-negative values. Thus, $\frac{\partial}{\partial R1}U1 = \frac{R2}{(R1+R2)^2} \cdot U3 \geq 0$, $\frac{\partial}{\partial R2}U1 = \frac{-R1}{(R1+R2)^2} \cdot U3 \leq 0$ and $\frac{\partial}{\partial U3}U1 = \frac{R1}{R1+R2} > 0$ holds and, therefore, the interval-based evaluation of the voltage divider rule can be defined as $U1 \in \left[\frac{R1left \cdot U3left}{R1left + R2right}, \frac{R1right \cdot U3right}{R1right + R2left}\right]$. By the same way, for each electrical law utilized for local propagation of qualitative values, we define its interval-based evaluation.

As explained, the SDSP method relies on the qualitative versions of electrical laws utilized for propagation of qualitative values. In the following, exemplarily, we show how the qualitative version of the *series compensation resistance rule* can be motivated.

The quantitative version of this rule is described by $S3 = R1 + R2$. As explained above, its interval-based evaluation is defined as $[S3left, S3right] \in [R1left + R2left, R1right + R2right]$. In order to define the qualitative version of this rule, first, the combination of normal values is considered. With respect to (1) *S3_normal = SCR(R1_normal, R2_normal)* must hold. According to the definition of the interval-based evaluation of $S3 = R1 + R2$ and the semantics of qualitative values (see Section 2), the quantitative landmarks S3min, S3max are specified in relation to R1min, R1max and R2min, R2max, i.e. $[S3min, S3max] = [R1min + R2min, R1max + R2max]$. Thus, the semantics of *S3_normal* is: $S3 \in [S3min, S3max]$ $= [R1min + R2min, R1max + R2max]$. However, what is the result of *SCR(R1_normal, R2_high)*? According to the interval-based evaluation of $S3 = R1 + R2$ and the semantics of the qualitative values of R1 and R2, $S3 \in [S3left, S3right] = (R1min + R2max, R1max + \infty)$ holds. Taking $0 < R1min < R1max < \infty$ and $0 < R2min < R2max < \infty$ into account, it is obvious that $0 < S3min < R1min + R2max < S3max < R1max + \infty$ holds. Thus, *SCR(R1_normal, R2_high)* = (*S3_normal* or *S3_high*) is valid (cf. Figure 2 and Table 3).

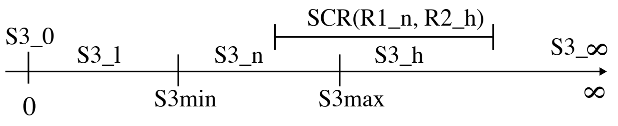

Figure 2: SCR(R1_n, R2_h) = S3_n or S3_h

The general principle behind the derivation methods for all qualitative operators is similar, i.e. the idea of the proof technique for the entries of these tables does neither depend on specific operations nor on specific qualitative values. Due to space limitations, only the results for qualitative *series compensation resistance rule* and *current transformation* are summarized in Table 3 and 4. The quantitative version of the *current transformation* is described by two different types of equations, one is $I2 = I23 - I12$. It is important to note that the *subtraction* of two positive intervals does not necessarily lead to a positive interval. Thus, the qualitative *current transformation (CT)* of two currents described by values of Type 1 leads to qualitative values of Type 2. Especially, *CT(0, 0)* has the qualitative values L, N or H as a result. Note that the set of values of Type 2 does not include any value that explicitly represents the quantitative value 0 (see Section 2).

R1 \ R2	0	l	n	h	∞
0	0	l	l / n	l / n / h	∞
l	l	l	l / n	l / n / h	∞
n	l / n	l / n	n	n / h	∞
h	l / n / h	l / n / h	n / h	h	∞
∞	∞	∞	∞	∞	∞

Table 3: Qual. series comp. resistance $S3 = SCR(R1, R2)$

I23 \ I12	0	l	n	h	∞
0	L / N / H	L / N / H	L / N	L	L∞
l	L / N / H	L / N / H	L / N	L	L∞
n	N / H	N / H	N	L / N	L∞
h	H	H	N / H	L / N / H	L∞
∞	H∞	H∞	H∞	H∞	?

Table 4: Qual. current transformation $I2 = CT(I23, I12)$

As we emphasized, assuming that a fault has occurred, well-known mathematical approaches for algebraic equation solving can compute correct qualitative values. Hence, we consider the SDSP method to be sound and complete if it computes exactly the same qualitative values. The problem with local propagation techniques is sometimes that intervals are widened, i.e. local propagation techniques are complete but possibly *unsound* results are generated. We cannot present the proof of soundness in detail in this paper but we briefly describe the main idea.

Let us assume, the qualitative value for a specific parameter C has to be determined for the case, that a resistor R has a qualitative value different from *R_normal*. Without restrictions we consider the case that the correct result for C is normal or higher (see I_ref, Figure 3).

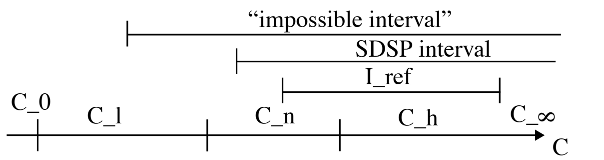

Figure 3: Sketch of the proof of soundness of the SDSP method

Since the SDSP method is complete, it determines an interval, that includes the interval I_ref. Under the single-fault assumption and the assumption that no faulty resistor is involved in a star-delta conversion, we show that the class of intervals represented by the “impossible interval” (see Figure 3) cannot be inferred by the SDSP method because the partial derivatives of the boundaries of the SDSP and the reference interval with respect to R are zero or of the same sign.

The soundness of the SDSP method is mainly achieved (i) by the introduction of the current and voltage divider rules (ii) by the qualitative one-step evaluation of electrical rules. If we neglected these two points, the SDSP method would become unsound. For a more detailed presentation of the proof of soundness and completeness we refer to (Milde, 1997).

3.3 SDSP-Analyzing the Field Regulator

In order to outline the strength of our approach we now show how to model and analyze the field regulator of our application domain. Faulty behavior of the field regulator is modeled by R5_low as an example.

The first step of the SDSP method is a star-delta conversion (see Figure 4) and a subsequent SP-reduction. As a result, the SP-tree shown in Figure 5 is obtained. Letters S and P indicate that nodes compensate series and parallel groupings, respectively.

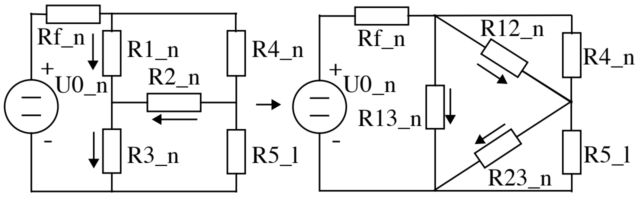

Figure 4: Resistor transformation

The second step of the SDSP method is local propagation of qualitative values. First, transformation resistance values are determined (see Figure 4). Second, qualitative values of compensation resistances, currents and voltages are propagated (see the labels of the arrows in Figure 5 and the corresponding legend).

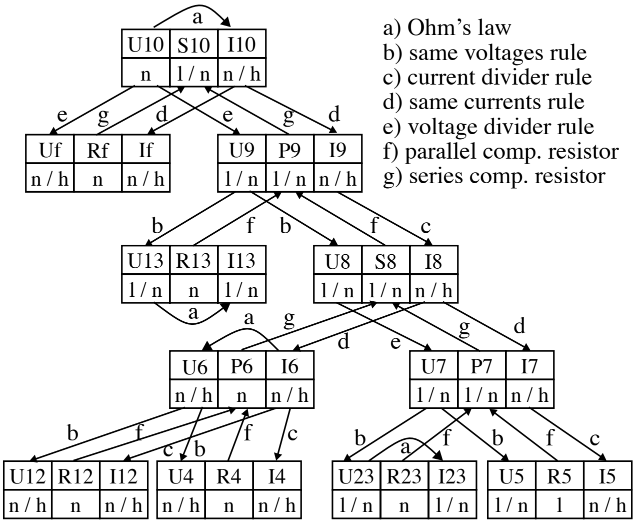

Figure 5: Propagation of qualitative values

The final step of the SDSP method is a voltage and current transformation in order to obtain the qualitative values of the original network. Due to space limitations, only I2 is discussed: $I2 = CT(I23, I12) = CT((l/n), (n/h)) = L/N$.

For instance, if we assume *R5_low* in order to model faulty behavior, the SDSP method will compute the *current* through R5 being equal or higher (in comparison to the faultless state). As a further result I2 is determined to be equal or lower (I2_L or I2_N). Note that, due to a current transformation operation, I2 is described by qualitative values of Type 2. In this case, a lower value might result in the inversion of the direction and, therefore, in an increase of the absolute value of the current. In order to evaluate the results concerning I2 for diagnostic purposes, the current of the field coil (bridge resistor) has to be measured by amount and by sign.

4. Conclusion

First of all, the SDSP method is applicable to almost arbitrary resistive networks that consist of one voltage source and an unlimited number of resistors. Thus, complex circuits can be handled and even in bridge circuits the algorithm computes the most restrictive set of qualitative values. As noted before, there are specific network topologies (see Section 3.1) that still cannot be handled.

Second, due to the well-known analogies between electricity, hydraulics and mechanics, the approach is not limited to the electrical domain.

Third, we introduce a set of qualitative values that represent deviations rather than signs. Due to the semantics of the qualitative values, it is possible to describe structural and non-structural faults and to distinguish their different effects on voltages and currents without utilizing quantitative parameter values.

Fourth, the complexity of the propagation of qualitative values of the SDSP method is linear with respect to the number of resistors in the circuit.

Fifth, the inference algorithm is sound and complete under certain assumptions mentioned above.

References

(Banzhaf, 1989) Banzhaf, W., *Computer-aided circuit analysis using SPICE*, Prentice Hall, Englewood Cliffs, New Jersey, 1989.

(Biasizzo and Novak, 1995) Biasizzo, A., Novak, F., *Model-Based Diagnosis of Analog Circuits with CLP(R),* Technical report CSD-TR-95-9, Jozef Stefan Institute, Ljubljana, Slovenia, 1995.

(Dague et al., 1990) Dague, P., Jhel, O., Tallibert, P., *an Interval Propagation and Conflict Recognition Engine for Diagnosing Continues Dynamic Systems*, in: Lecture Notes in AI, 462, September 1990.

(Mauss and Neumann, 1996) Mauss, J., Neumann, B., *How to Guide Qualitative Reasoning about Electrical Circuits by Series-Parallel Trees*, in: Proc. QR'96, 10th Int. Workshop on QR, Stanford, CA, 1996.

(Milde, 1997) Milde, H., *Untersuchung der SDSP-Analyse auf Vollständigkeit und Korrektheit,*LKI-Report LKI-M-97/3, Artificial Intelligence Lab., Univ. Hamburg, 1997.

(Mohamed and Marzouki, 1996) Mohamed, F., Marzouki, M., *Test and Diagnosis of Analog Circuits: When Fuzziness can Lead to Accuracy*, in: Journal of Electronic Testing: Theory and Applications, 9, 1-15 (1996), 1996 Kluwer Academic Publishers, Boston.

(Struss, 1990) Struss, P., *Problems of Interval-Based Qualitative Reasoning*, in: Readings in Qualitative Reasoning about Physical Systems, Weld, D., de Kleer, J. (Eds.), Morgan Kaufmann, 1990

(Struss et al., 1995) . Struss, P., Malik, A., Sachenbacher, M., *Qualitative Modelling is the Key.* , in: Proc. DX-95, 6th Int. Workshop on Principles of Diagnosis, 1995