Resistive Networks Revisited: Exploitation of Network Structures and Qualitative Reasoning about Deviations is the Key

Lothar Hotz, Heiko Milde, Ralf Möller, Bernd Neumann
Laboratory for Artificial Intelligence, University of Hamburg

Abstract

For diagnostic purposes, analog circuits may be qualitatively modeled as resistive networks. We demonstrate that common approaches to this task show certain weaknesses because of their sign-based qualitative values. In order to overcome these deficiencies, we first introduce qualitative deviation values with a semantics that enables us to model different classes of faults arising in analog circuits. The qualitative values adequately describe different effects that faults may have. Then we present a sound and complete inference algorithm for computing these effects using qualitative operators and local propagation techniques.

1. Introduction

In the past, many different approaches to model-based diagnosis of analog circuits have been published. For instance, if the circuit parameters can be described by crisp quantitative values, a linear network can be analyzed by existing tools such as SPICE [Banzhaf, 1989] or systems based on CLP(*R*) (e.g. [Biasizzo and Novak, 1995]). In order to cope with tolerances and inaccuracies, the DIANA system [Dague et al., 1990] uses quantitative *intervals* to describe network parameters. The FLAMES system [Mohamed and Marzouki, 1996] proposes *fuzzy* intervals to describe inaccuracies more adequately.

While these systems can be used to simulate a large class of analog circuits by exploiting detailed component models, [Struss et al., 1995] argue that for diagnostic purposes more abstract models are advantageous. In particular, resistive networks with qualitative parameter values have been investigated in the literature.

For instance, adhering to the no-function-in-structure principle, the Connectivity Method [Struss et al., 1995] basically propagates qualitative information that encodes which port of a circuit component is connected to source and sink (direction and kind of resistance, i.e. zero resistance, finite resistance or, in case of a fault, infinite resistance). However, not all kinds of circuits can be handled adequately (e.g. bridge circuit topologies). In order to overcome these deficiencies, [Mauss and Neumann, 1996] have developed a qualitative method to analyze resistive networks by exploiting the *structure* of networks. The so-called SPS method explicitly represents a network's series-parallel-star structure as a tree (sps-tree). As a result of the network analysis, for all currents and voltages, *sign-based* qualitative values are determined. In our opinion the Connectivity Method and the SPS method focus on the detection of *structural faults*, e.g. broken wires or comparable component faults such as blown light bulbs etc. They do not address, however, several diagnosis tasks which may arise in resistive networks:

- non-structural faults such as slight deviations from normal behavior,
- deviative effects of non-structural and structural faults,
- specific circuit topologies,
- dealing with abstractions in diagnosis models,
- dealing with variants.

This paper presents a qualitative method for these topics. The problems solved by the approach are explained with an application example which deals with electricity-powered fork-lifts. In particular, we consider a field regulator that is a subcomponent of a motor. A schematic diagram of the field regulator circuit is presented in Figure 1. The components shown in the figure are abstractions of the real physical components. For instance, the control switches T1 to T4 are actually implemented with transistors and diodes but, for diagnostic purposes, such a fine-grained representation is not required.

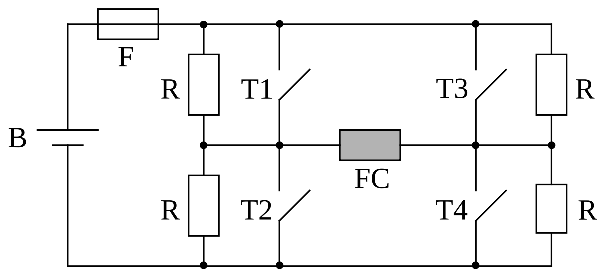

Figure 1: Field regulator (resistors (R), fuse (F), battery (B), field coil (FC), controlled switches (T1 to T4)). In this figure the electronic circuit for controlling the switches has been omitted.

In our application we focus on non-structural faults such as slight deviations from normal behavior, e.g. increased resistance values. Faults of this kind can neither be *modeled* by the SPS method nor by the Connectivity Method because of the sign-based qualitative values used by these methods. In analog circuits the occurrence of a fault effects all currents and voltages, i.e. the absolute values of parameters change. However, in most cases, the parameters do not change in their signs (or reach a certain limit). Thus, it is hardly possible to adequately derive these fault *effects* using sign-based qualitative values. Furthermore, bridge circuit topologies are relevant in our domain (see Figure 1). These circuits pose a special problem for qualitative approaches because the direction of the current through the bridge resistor usually depends on the exact quantitative values of the component parameters.

Since the components of our model are abstractions of real components, *quantitative* modeling systems (see above) are not appropriate, either. Note that, in general, it is difficult to derive useful quantitative values for abstract components. In addition, we also have to deal with the "variants' dilemma" [Struss et al., 1995]. This means that a certain model of the field regulator should cover several variants of this device. Variants differ only slightly concerning their values of component parameters. Thus, in principle, qualitative methods are preferable.

Based on the SPS method we introduce a new qualitative approach for reasoning about analog circuits for diagnostic purposes. The main features of our method are:

- The qualitative values represent deviations as well as sign information. With deviations we can describe non-structural faults such as "resistance too high" as well as structural faults such as blown light bulbs ("resistance too high and infinite") even in bridge circuit topologies.
- The semantics of the qualitative values is grounded on the quantitative nature of landmarks and their algebraic relations (e.g. order relations). This way we can show soundness and completeness of the derivation algorithm for qualitative reasoning (cf. also [Struss, 1990]).
- We show that it is not necessary to specify the absolute quantitative values of landmarks. Considering the order relation between landmarks allows us to deal with abstract circuit components and provides a basis for dealing with the "variants' dilemma". The method pre-

sented in this paper derives simulation results that are sufficient for fault discrimination in a diagnosis application.

The key idea of our approach is (i) to derive a set of qualitative values (deviation values) to adequately describe faults and their effects and (ii) to define qualitative operators to propagate these values in order to simulate circuit behavior. The paper is structured as follows. Qualitative values for describing deviations are introduced in Section 2. In Section 3 we describe a qualitative calculus simulating circuit behavior based on deviations. The algorithm is shown to be sound and complete. Section 4 points out the main achievements in a conclusion.

2 Qualitative values

In a resistive network, there are currents and voltages whose directions can be determined by the structure of the network. The values of these currents and voltages as well as the values of resistances can be a set of qualitative values that cover the extended *positive* real number line $[0, \infty]$ (Type 1). For some currents and voltages, the directions are not determined by the network structure. Thus, we also need qualitative values that cover the *whole* extended real number line $[-\infty, \infty]$ (Type 2). As long as only qualitative deviation values of Type 1 can be used, a qualitative calculus can derive more concise results (i.e. sign information is retained). Distinguishing between two types of qualitative value sets is not merely a syntactic criterion but sharpens the reasoning about effects in resistive networks.

2.1 Qualitative values, Type 1

The qualitative values of Type 1 and their semantics are shown in Table 1.

qualitative value	abbreviation	semantics
A_low_0	A_0	$A \in [0, 0]$
A_low	A_l	$A \in (0, Amin)$
A_normal	A_n	$A \in [Amin, Amax]$
A_high	A_h	$A \in (Amax, \infty)$
A_high_inf	A_∞	$A \in [\infty, \infty]$

Table 1: Qualitative values of Type 1

Although the landmark values in the semantics definition are not specified as fixed quantitative values, we do rely on the order between landmarks: $0 < Amin < Amax < \infty$. The qualitative value *A_normal* represents an interval that encodes the range of parameter A in the faultless state. The other qualitative values describe deviations from the faultless state. We distinguish between extreme deviation values *A_low_0* and *A_high_inf* and non-extreme deviation values *A_low* and *A_high*.

Describing resistances (R) the qualitative values of Type 1 can be used to model structural as well as non-structural faults. On the one hand, structural faults such as short circuits and broken wires can be described by the extreme deviations *R_low_0* and *R_high_inf*, respectively. On the other hand, non-structural faults such as partial short circuits in coils and corroded wiring points can be modeled by the qualitative values *R_low* and *R_high*, respectively. The faultless state of a component is represented by the qualitative value *R_normal* which represents an interval. This enables us to be tolerant with respect to differential deviations such as physical tolerances and temperature drifts.

Voltages (U) and currents (I) can also be described by qualitative values of Type 1 if their directions are determined by the structure of the network. Again, the qualitative value *U/I_normal* characterizes the faultless state. Structural faults can result in extreme deviation values, e.g. there is no current (*I_low_0*) through a broken wire, as well as in non-extreme deviations, e.g. there is higher current (*I_high*) in parallel paths of a broken wire. Note that in sign-based approaches (e.g. [Mauss and Neumann, 1996]) effects like these cannot be modeled. Non-structural faults mostly lead to non-extreme deviations. For instance, there is lower current (*I_low*) through corroded wiring points.

2.2 Qualitative values, Type 2

The qualitative values of Type 2 and their semantics are shown in Table 2. We use capital letters to distinguish the different types.

qualitative value	abbreviation	semantics
A_Low_neg_inf	A_L_∞	$A \in [-\infty, -\infty]$
A_Low	A_L	$A \in (-\infty, Amin)$
A_Normal	A_N	$A \in [Amin, Amax]$
A_High	A_H	$A \in (Amax, \infty)$
A_High_inf	A_H_∞	$A \in [\infty, \infty]$

Table 2: Qualitative values of Type 2

Again, we emphasize the order relation between the landmarks: $-\infty < Amin < Amax < \infty$. The qualitative value *U/I_Normal* describes the faultless state, but it does not indicate whether the interval is located to the left or to the right of the zero point of the extended real number line. *U/I_Low_neg_inf* and *U/I_High_inf* describe extreme deviations from the faultless state. *U/I_Low* and *U/I_High* specify non-extreme deviations.

A challenging task for diagnosis systems based on fault models is to derive the effects of faults on network parameters, i.e. starting at the point where one or even more resistances are described by qualitative values different from *R_normal,* currents and voltages at metering points have to be determined.

3. SDSP-Analysis

A resistive network can be described by a system of linear equations based on Kirchhoff's and Ohm's laws. Assuming that a fault has occurred, this system of equations can be exploited in order to determine qualitative values for voltages and currents. Since the qualitative values represent intervals, the system of equations can be algebraically solved. As a result, the functional relationship between a certain voltage (current) on the one hand and the resistances and the voltage source on the other hand can be obtained. These functional relationships can vary widely in complexity depending on the specific network structures. In order to compute the qualitative values for a voltage (current), the signs of partial derivatives of the above-mentioned functional relationship have to be determined. Moreover, symbolic expressions describing interval boundaries have to be ordered by size. Although possible, this process seems to be very complicated because the symbolic expressions being involved can be extremely complex. One approach to simplify this under specific circumstances has been published by [Mauss and Neumann, 1996]. The main advantage of the SPS method is that the network is described by a set of component-oriented local equations which can be solved step by step. This means that there is always at least one equation that can be directly solved. The equations are organized in a SP-tree which directly relates corresponding variables and, therefore, solving the set of equations means local propagation of values guided by the SP-tree. The SPS method currently is a sign-based approach for analyzing resistive networks. In the introduction we have seen that with sign-based approaches not all faults and their different effects can be modeled. Therefore, we adapt the SPS method to the deviation-based qualitative values introduced in Section 2 and demonstrate that rules known from electrical engineering can be interpreted in such a way that a local propagation algorithm can also be defined for deviation-based qualitative values.

3.1 Qualitative analysis of resistive networks

Our approach to the qualitative analysis of a resistive network consists of two main steps. We first describe these steps using the quantitative interpretation of electrical laws in order to show that we utilize a limited number of different types of equations. This is important since each type of equation will give rise to a qualitative operator in the qualitative version of the analysis.

1. SDSP transformation:

The circuit transformation consists of star-delta transformation and series-parallel reductions which generate an SP-tree whose nodes are attached with equations that are used to compute values of network parameters (transformation resistances, series-parallel compensation resistances, current and voltages). This step is carried out once. Similarly to the SPS method, the resulting structure will be used to simulate different kinds of faults. The SP-tree is an explicit representation of the structure of the network.

If the network is not series-parallel reducible (SP-reducible) in the first place, star-delta transformations can be performed. As a result, the transformed network consists exclusively of series and parallel groupings. The star-delta conversion for circuit transformations is shown in Figure 2.

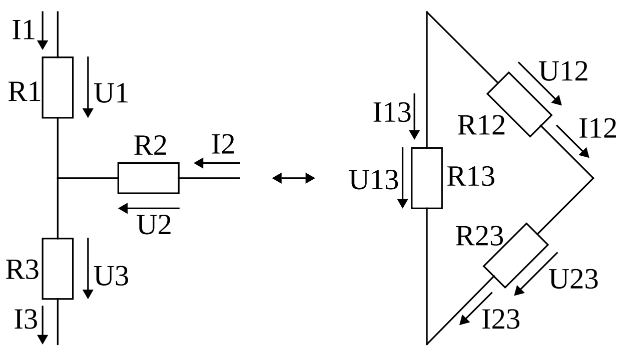

Figure 2: Star-delta conversion

The network conversion consists of resistance, current and voltage transformations. Resistance transformations are described by the following quantitative equations (1) to (6).

$$(1)\quad R1 = \frac{1}{\frac{1}{R13} + \frac{1}{R12} + R32 \cdot \frac{1}{R12 \cdot R13}}$$

$$(2)\quad R12 = R1 + R2 + R1 \cdot R2 \cdot \frac{1}{R3}$$

$$(3)\quad R2 = \frac{1}{\frac{1}{R12} + \frac{1}{R32} + R13 \cdot \frac{1}{R12 \cdot R13}}$$

$$(4)\quad R13 = R1 + R3 + R1 \cdot R3 \cdot \frac{1}{R2}$$

(5) $R3 = \frac{1}{\frac{1}{R13} + \frac{1}{R32} + R12 \cdot \frac{1}{R13 \cdot R32}}$ (6) $R23 = R2 + R3 + R2 \cdot R3 \cdot \frac{1}{R1}$

Current transformations are described by the equations (7) to (9).

(7) $I1 = I12 + I13$ (8) $I2 = I23 - I12$ (9) $I3 = I13 + I23$

Voltage transformations are described by the equations (10) to (12).

(10) $U12 = U1 - U2$ (11) $U13 = U1 + U3$ (12) $U23 = U2 + U3$

As a difference to the SPS method we would like to emphasize that we use star as well as delta conversions, (hence the name of our method: SDSP method). This is advantageous because, in comparison to the SPS method, new classes of network topologies can be treated. As a further difference, we restrict stars and deltas to be transformed to those with three edges - with the purpose to obtain a fixed number of equation types. And hence a fixed set of qualitative transformation operators. Thus, the number of equations that are exploited by the SDSP method is limited. As a disadvantage of this restriction to stars and deltas with three edges, we admit that there are some networks that cannot be treated (e.g. networks consisting exclusively of four-edge stars without any delta transformations applicable). According to our experiences, these networks are hardly relevant in practice.

In order to present the naming conventions, the schemes of series and parallel groupings of resistors are shown in Figure 3.

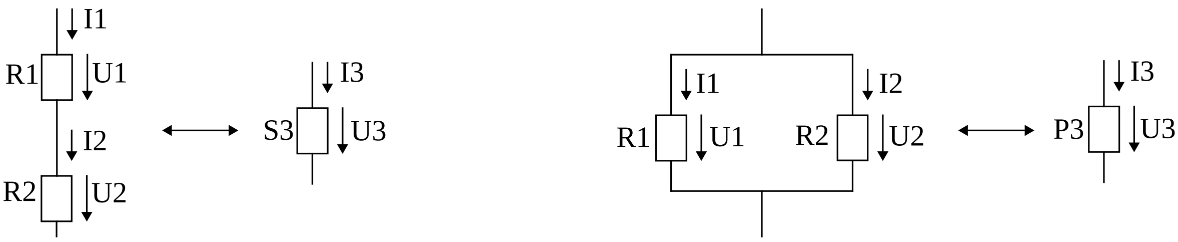

Figure 3: Series and parallel grouping

Series groupings of resistors are described by equations (13) to (15).

(13) $S3 = R1 + R2$, serial compensation resistor

(14) $I1 = I2 = I3$, same currents rule

(15) $U1 = \frac{R1}{R1 + R2} \cdot U3$, voltage divider rule

Parallel grouping of resistors are described by equations (16) to (18).

(16) $P3 = \frac{1}{\frac{1}{R1} + \frac{1}{R2}}$, parallel compensation resistor

(17) $U1 = U2 = U3$, same voltages rule

(18) $I1 = \frac{R2}{R1 + R2} \cdot I3$, current divider rule

2. Local propagation of qualitative values:

The second step consists of local propagation of qualitative values in the SP-tree in order to simulate circuit behavior (i.e. the step can be carried out for each of the supplied fault models).

First, by exploiting equation (1) to (6) the values of transformation resistances are determined. Note, that the special form of the equations (1) to (6) has been derived in order to deal with the selection problem [Struss, 1990], i.e. we do not utilize $R1 = \frac{R13 \cdot R32}{R12 + R13 + R32}$ which is obviously algebraically equivalent to equation (1). The resistance values of transformed resistors are determined with equations (1) to (6). Since resistances are described by qualitative values of Type 1, the qualitative versions of equations (1) to (6) have to be defined for values of Type 1.

Second, by exploiting equations (13) and (16), values of resistances are propagated from the leaves of the SP-tree to its root. Since resistances are described by qualitative values of Type 1, the qualitative versions of these equations have to be defined on values of Type 1. Note the special form of equation (16) which is specifically chosen in order to handle the selection problem. We do not utilize $P3 = \frac{R1 \cdot R2}{R1 + R2}$ which is algebraically equivalent to equation (16).

Third, values for currents and voltages are propagated from the root of the SP-tree to its leaves by evaluating equation (14), (15), (17) and (18). As a result, values for each current and each voltage of the transformed network are obtained. In the transformed network the directions of currents and voltages are determined by the network's structure. Thus, currents and voltages are described by qualitative values of Type 1 and, therefore, the qualitative version of equation (14), (15), (17) and (18) have to be defined on values of Type 1 (see below). This step of the network analysis is an extension of the SPS method since we exploit an extended set of electrical laws, i.e. current divider and the voltage divider rules are added.

One could argue that these rules violate the no-function-in-structure principle because they are only applicable if certain groupings of resistors are concerned. Nevertheless we do not hesitate to exploit these rules, because using them does not imply any limitations on the applicability of our approach, i.e. the resistors can still be arbitrarily connected. Furthermore, we show that these two rules are required by the propagation algorithm (see the comments on soundness and completeness in Section 4). Obviously, the form of the equations (15) and (18) indicates that qualitative versions of the current divider and voltage divider rules suffer from the selection problem which cannot be overcome by choosing specific equation transformations. The qualitative versions of these two rules are explicitly defined in Section 3.2.

Fourth, values of voltages and currents of the original network are determined by exploiting equations (7) to (12). Up to this point of the network analysis, currents and voltages are described by qualitative values of Type 1. Thus, the qualitative version of these rules have to be defined on values of Type 1. The *subtraction* of two qualitative values of Type 1 (see the equations (8) and (10)) leads to qualitative values of Type 2 because the *subtraction* of two positive intervals does not necessarily lead to a positive interval. Furthermore, values of voltages and currents are obtained by evaluating Ohm's law. Since Ohm's law combines resistance values on the one hand and the values of currents and voltages on the other hand, the qualitative version of Ohm's law has to be defined for the combination of two qualitative values, one from Type 1 one from Type 2.

In order to define the qualitative versions of the electrical laws mentioned above, the qualitative versions of *addition*, *multiplication*, *reciprocal*, *subtraction* and the current divider and the voltage divider rule have to be defined on values of Type 1. Moreover, the qualitative version of the *multiplication* has to be defined for two different values, one from Type 1 the other from Type 2. In the next section the definition of these operators are given. The qualitative *equal* operator is not mentioned because it is trivial.

3.2 Combining qualitative values during SP-tree propagation

The SDSP method relies on the definitions of the above mentioned qualitative operators. These definitions are based on the following three features. We use uppercase letters to describe qualitative operators, e.g. ADD means qualitative *addition*.

(I) The qualitative values *A_normal* and *A_Normal* represent the faultless state. Therefore, any parameter A has the qualitative value A*_normal (A_Normal)* if its value is determined from parameters that, in turn, have the qualitative values A*_normal (A_Normal)*. E.g., ADD(*R1_normal, R2_ normal) = S3_normal* with S3 being the compensation resistor of the series grouping of R1 and R2.

(II) The qualitative values of the SDSP method have a clear semantics, e.g. *A_normal* <-> $A \in [Amin, Amax]$. As noted before, we emphasize that the qualitative values represent symbolic intervals whose boundaries are not quantitatively specified, but they are ordered, e.g. $0 < Amin < Amax < \infty$.

(III) Due to the semantics of qualitative values, the interval calculus presented by [Struss, 1990] can be utilized for defining qualitative operators. In addition, we use the well known reciprocal of an interval with the definitions $\frac{1}{[0, 0]} = [\infty, \infty]$ and $\frac{1}{[\infty, \infty]} = [0, 0]$. Exploiting the current divider and the voltage divider rule by applying the elementary operations for interval arithmetic, might lead to the selection problem. For instance, in equation (15) and (18) the resistances R1 and R2 appear in the numerator as well as in the denominator. In order to avoid unnecessary large intervals we define the one-step evaluation of these two rules for intervals as follows.

If $R1 \in [R1left, R1right]$ and $R2 \in [R2left, R2right]$ and $U3 \in [U3left, U3right]$ and $U1 = \frac{R1}{R1 + R2} \cdot U3$ then

$$U1 \in [U1left, U1right] = \left[\frac{R1}{R1 + R2} \cdot U3 \Bigg|_{min}, \frac{R1}{R1 + R2} \cdot U3 \Bigg|_{max} \right]$$ holds.

Note that $U3, R1, R2 \geq 0$ because the current and voltage divider rules are only applied when SP-reducible networks are considered. In this case, all currents and voltages have non-negative values. Thus, the equations

$\frac{\partial}{\partial R1} U1 = \frac{R2}{(R1 + R2)^2} \cdot U3 \geq 0$ and $\frac{\partial}{\partial R2} U1 = \frac{-R1}{(R1 + R2)^2} \cdot U3 \leq 0$ and

$\frac{\partial}{\partial U3}U1 = \frac{R1}{R1+R2} > 0$ hold and, therefore, the interval-based evaluation of the voltage divider rule can be defined as follows.

$$[U1left, U1right] = \left[\frac{R1left}{R1left + R2right} \cdot U3left, \frac{R1right}{R1right + R2left} \cdot U3right\right].$$

By the same way, we define the interval-based current divider rule.

$$[I1left, I1right] = \left[\frac{R2left}{R1left + R2right} \cdot I3left, \frac{R2right}{R1right + R2left} \cdot I3right\right]$$

As explained in Section 3.1, the SDSP method relies on the qualitative versions of the arithmetical operations *addition*, *multiplication*, *reciprocal* and *subtraction* of qualitative values of Type 1 as well as on the *multiplication* of two qualitative values, one from Type 1 the other from Type 2. The definition of these operations on qualitative values is given in Tables 3 to 7. Based on the derivations in (III), the application of the current and voltage divider rules to qualitative values of Type 1 is presented in Table 8.

For all operations a specific combination table is defined. The tables specify the composition of normal as well as non-normal qualitative values. In the following we explain the entries of the composition tables used in the SDSP method. Exemplarily, we show how the results of the *addition* of two qualitative values of Type 1 can be motivated.

In order to define the qualitative addition $C = ADD(A, B)$ of two values of Type 1, first, the combination of normal values is considered. According to the semantics of qualitative values (see Section 2), we have to specify the quantitative landmarks Cmin, Cmax in relation to Amin, Amax and Bmin, Bmax. With respect to (I)

C_normal = ADD(A_normal, B_normal)

must hold. In Section 2, the semantics of the qualitative values is given. Thus,

$$[Cmin, Cmax] = [Amin, Amax] + [Bmin, Bmax]$$

is valid. According to [Struss, 1990]:

$$[Cmin, Cmax] = [Amin + Bmin, Amax + Bmax]$$

The semantics of *C_normal* is:

C_normal <-> $C \in [Cmin, Cmax] = [Amin + Bmin, Amax + Bmax]$

However, what is the result of *ADD(A_normal, B_high)*? According to the semantics of qualitative values A and B:

$$[Amin, Amax] + (Bmax, \infty) = (Amin + Bmax, Amax + \infty)$$

Taking $0 < Amin < Amax < \infty$ and $0 < Bmin < Bmax < \infty$ into account, it is obvious that

$$0 < Cmin < Amin + Bmax < Cmax < Amax + \infty$$

holds. Thus,

ADD(A_normal, B_high) = (*C_normal* or *C_high*)

is valid (cf. Figure 4 and Table 3).

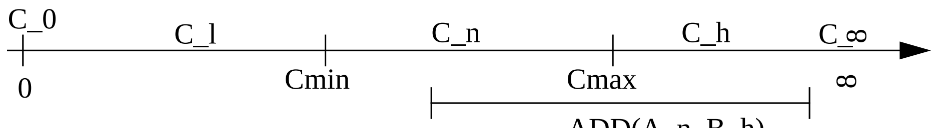

Figure 4: ADD(A_n, B_h) = C_n or C_h

The general principle behind the derivation methods for all qualitative operators is similar, i.e. the idea of the proof technique for the entries of these tables does neither depend on specific operations nor on specific qualitative values. Due to space limitations, the results are summarized in Table 3 to Table 8. Slashes (/) mean logical “or” and question marks indicate that corresponding operations are undefined. We use lowercase (uppercase) letters for parameters in the operator definitions in order to indicate that the operators are defined on parameters that are described by qualitative values of Type 1 (Type 2).

The Tables 3 to 6 define the elementary operations on the qualitative values of Type 1.

A | B	0	l	n	h	∞
0	0	l	l / n	l / n / h	∞
l	l	l	l / n	l / n / h	∞
n	l / n	l / n	n	n / h	∞
h	l / n / h	l / n / h	n / h	h	∞
∞	∞	∞	∞	∞	∞

Table 3: Qualitative addition $c = ADD(a, b)$ of values of Type 1

A | B	0	l	n	h	∞
0	0	0	0	0	?
l	0	l	l / n	l / n / h	∞
n	0	l / n	n	n / h	∞
h	0	l / n / h	n / h	h	∞
∞	?	∞	∞	∞	∞

Table 4: Qualitative multiplication $c = MULT(a, b)$ of values of Type 1

A	0	l	n	h	∞
	∞	h	n	l	0

Table 5: Qualitative reciprocal $c = RECIP(a)$ of values of Type 1

It is important to note that the *subtraction* of two positive intervals does not necessarily lead to a positive interval. Thus, the qualitative *subtraction* of two values of Type 1 leads to qualitative values of Type 2 (see Table 6). Especially, SUB(0, 0) has the qualitative values L, N or H as a result. Note that the set of values of Type 2 does not include any value that explicitly represents the quantitative value 0 (see Section 2).

A | B	0	l	n	h	∞
0	L / N / H	L / N / H	L / N	L	L∞
l	L / N / H	L / N / H	L / N	L	L∞
n	N / H	N / H	N	L / N	L∞
h	H	H	N / H	L / N / H	L∞
∞	H∞	H∞	H∞	H∞	?

Table 6: Qualitative subtraction $C = SUB(a, b)$ of values of Type 1

The qualitative *multiplication* of two values, one from Type 1 the other from Type 2, is defined in Table 7. Some combinations are undefined (question marks) because each of the qualitative values L, N, H may represent an interval that contains the quantitative value 0 and the *multiplication* of ∞ and the value 0 is not defined.

A | B	0	l	n	h	∞
L∞	?	L∞	L∞	L∞	L∞
L	L / N / H	L / N / H	L / N	L / N / H	?
N	L / N / H	L / N / H	N	L / N / H	?
H	L / N / H	L / N / H	N / H	L / N / H	?
H∞	?	H∞	H∞	H∞	H∞

Table 7: Qualitative multiplication $C = MULT(A, b)$ of values of different types.

The definition of the qualitative current divider and the voltage divider rule on values of Type 1 is shown in Table 8.

R1 | R2 or R2 | R1	0	l	n	h	∞	U3 or I3
0	? ? ?	0 0 0	0 0 0	0 0 0	0 0 0	l n h
l	l / n / h n / h h	l / n / h l / n / h l / n / h	l / n l / n l / n / h	l l / n l / n / h	0 0 0	l n h
n	l / n / h n / h h	l / n / h n / h n / h	l / n n n / h	l / n l / n l / n / h	0 0 0	l n h
h	l / n / h n / h h	l / n / h n / h h	l / n / h n / h n / h	l / n / h l / n / h l / n / h	0 0 0	l n h
∞	l / n / h n / h h	l / n / h n / h h	l / n / h n / h h	l / n / h n / h h	? ? ?	l n h

Table 8: Qualitative voltage divider rule $u1 = VDR(r1, r2, u3)$
and current divider rule $i1 = CDR(r1, r2, i3)$

3.3 Modeling and SDSP-Analyzing the field regulator

In order to outline the strength of our approach we now show how to model and analyze the field regulator of our application domain. A parallel grouping of a resistor and a switch (see Figure 1) is represented by only one resistor in order to simplify the model, (Figure 5). The field coil and the fuse are modeled as resistors as well (Rf and R2). The battery is described by a voltage source (U0). The resistor R5 models a faulty behavior of the field regulator because it has the qualitative value *R5_low*. The network to be analyzed is a bridge circuit.

The first step of the SDSP method is a star-delta conversion (including corresponding resistance transformations) and a subsequent SP-reduction. As a result, the SP-tree is obtained.

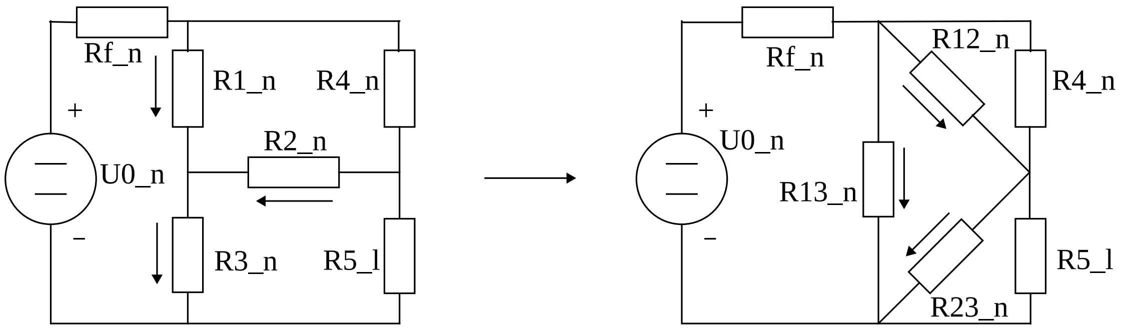

Figure 5 : Resistor transformation

For the circuit in Figure 5 the SP-tree is shown in Figure 6. R5 and R23 build a parallel grouping and therefore they are the child nodes of the node P7. The letter P indicates that the node 7 compensates a parallel grouping. Series groupings are represented by nodes marked with the letter S.

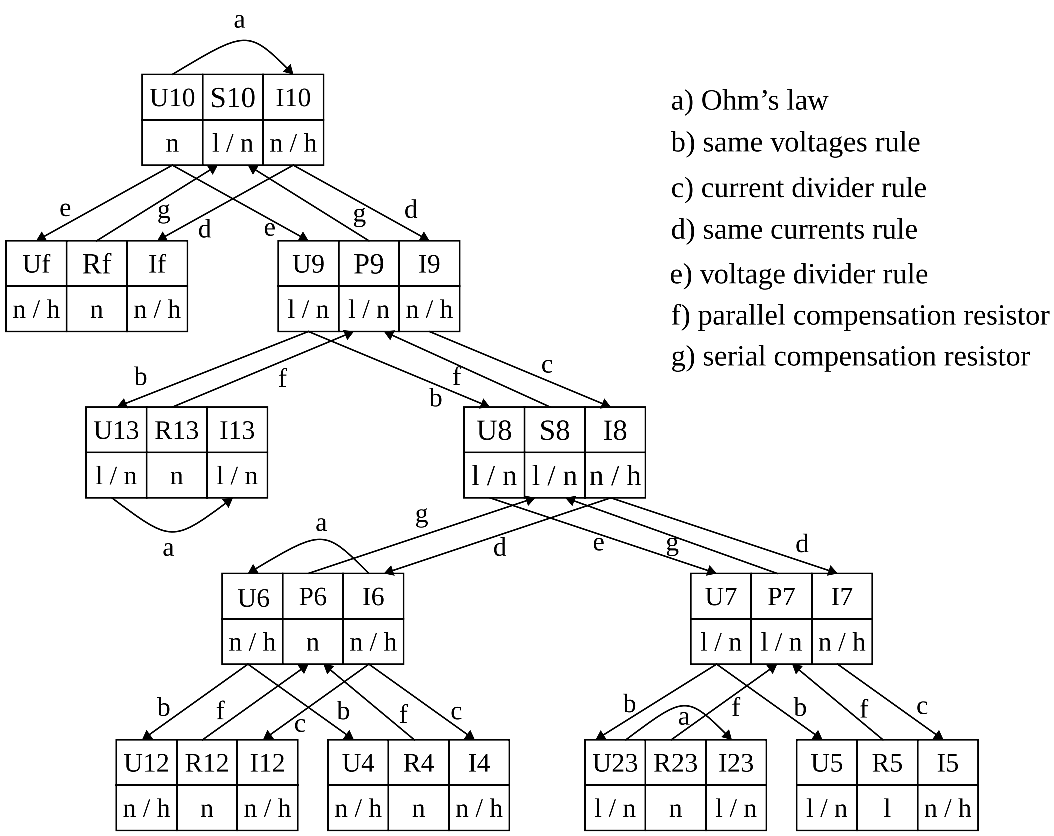

Figure 6: Propagation of qualitative values

The second step of the SDSP method is local propagation of qualitative values.

First, the transformation resistance values are determined. Since the involved resistances of the original network have the qualitative value *Ri_normal*, the transformed resistances are described by *normal*, too (see Figure 5).

Second, the qualitative values of the compensation resistances are determined. The leaves of the SP-tree show the qualitative values that are presented in Figure 5. These values are propagated to the root (S10) of the tree using the parallel and the series compensation resistor rule. The value describing P7 is derived by applying the parallel compensation resistor rule.

Third, the qualitative values of currents and voltages are computed. The voltage source is applied to the whole network and, thus, it is applied to the node S10 in the SP-tree and, therefore, *U10_normal* holds. Qualitative values of currents and voltages are propagated top-down by applying the propagation rules (see the labels of the arrows in Figure 6 and the corresponding legend).

The final step of the SDSP method is a voltage and current transformation in order to obtain the qualitative values of the original network.

$$I1 = ADD(I12, I13) = ADD((n/h), (l/n)) = l/n/h$$

$$I2 = SUB(I23, I12) = SUB((l/n), (n/h)) = L/N$$

$$I3 = ADD(I13, I23) = ADD((l/n), (l/n)) = l/n$$

$$U1 = MULT(R1, I1) = MULT(n, (l/n/h)) = l/n/h$$

$$U2 = MULT(R2, I2) = MULT(n, (L, N)) = L/N$$

$$U3 = MULT(R3, I3) = MULT(n, (l/n)) = l/n$$

For instance, if we assume, the qualitative *resistance* value of resistor R5 is *R5_low* in order to model faulty behavior, the SDSP method will compute the *current* through R5 being equal or higher (in comparison to the faultless state). More surprisingly, for R1, it cannot be derived whether the *current* is higher, lower or equal. In other words: The current's deviation from the faultless state depends on the exact quantitative values of resistances in the circuit. As a further result of the application of the SDSP method, I2 is determined to be equal or lower (I2_L or I2_N). Note that, due to a subtraction operation (cf. equation (8)), I2 is described by qualitative values of Type 2. In this case, a lower value might result in the inversion of the direction and, therefore, in an increase of the absolute value of the current. In order to evaluate the results concerning I2 for diagnostic purposes, the current of the field coil (bridge resistor) has to be measured by amount and by sign.

4 Conclusion

First of all, the SDSP method is applicable to almost arbitrary resistive networks that consist of one voltage source and an unlimited number of resistors. Thus, complex circuits can be handled and even in bridge circuits the algorithm computes the most restrictive set of qualitative values. As noted before, there are specific network topologies (see Section 3.1) that still cannot be handled.

Second, due to the-well known analogies between electricity, hydraulics and mechanics, the approach is not limited to the electrical domain.

Third, we introduce a set of qualitative values that represent deviations rather than signs. Due to the semantics of the qualitative values, it is possible to describe structural and non-structural faults and to distinguish their different effects on voltages and currents without utilizing quantitative parameter values.

Fourth, the qualitative versions of electrical rules are specifically chosen to avoid the selection problem. The inference algorithm is sound and complete under the single-fault assumption and the assumption that no faulty resistor is involved in a star-delta conversion. The latter assumption is related to the single-fault assumption because with a bridge resistor being non-normal the star-delta conversion would generate three compensation resistors with non-normal behavior. The complexity of the local propagation of qualitative values of the SDSP method is linear with respect to the number of resistors in the circuit.

[Struss, 1990] has defined the notion of soundness and completeness of a qualitative inference system based on its quantitative counterpart. As we have seen, not in all applications, quantitative values are available. Therefore, we have defined a semantics of qualitative deviation values based on order information alone. Thus, in our setting, the SDSP method cannot directly be compared to quantitative network analysis methods. As we emphasized, a resistive network can be described by a set of global linear equations that are based on Ohm's and Kirchhoff's laws. There are well-known mathematical approaches for algebraic equation solving that compute correct results, i.e. the corresponding derivation algorithms are sound and complete. However, as we have argued, local propagation methods such as SDSP are much simpler. The problem with local propagation techniques is that intervals are widened, i.e. possibly unsound results are generated due to uncorrelated local propagation operations. In qualitative approaches, intervals are represented as disjunctions of qualitative values. In some cases, widening of intervals even results in incorrect qualitative values in disjunctions being computed by the propagation algorithm. Thus, the proof for the soundness of the SDSP method must show that this cannot happen for all kinds of network structures being treated. The completeness proof must show that the intervals are not restricted too much, i.e. the interval disjunctions are not "too small". Completeness can easily be shown by induction over the involved propagation rules while soundness is more difficult to show. We cannot present this proof in detail in this paper due to lack of space (we refer to [Milde, 1997]), but we briefly describe the main idea. Let us assume, the qualitative value for a specific parameter C has to be determined for the case, that a resistor R has a qualitative value different from *R_normal*. Without restrictions we consider the case that the result for C is normal or higher (see Figure 7). If quantitative values were available, the left margin of the reference interval I_ref lies in the range of C_n and the right margin lies in the range of C_h. Since only order information is available, we do not know the exact position of I_ref.

Since the SDSP method is complete, during network analysis it determines an interval for parameter C, that includes the interval I_ref (see Figure 7). Under the assumptions mentioned above, we show that the class of intervals represented by the "impossible interval" (see Figure 7) cannot be inferred by the SDSP method. Exploiting the single-fault assumption we rule out these intervals by showing that the partial derivatives of the boundaries of the SDSP and the reference interval with respect to R are zero or of the same sign. This proof is based on the exact set of propagation operators that we have defined in the previous section. Especially, the soundness of the SDSP method is achieved (i) by the introduction of the current and voltage divider rules (ii) by their qualitative one-step evaluation.

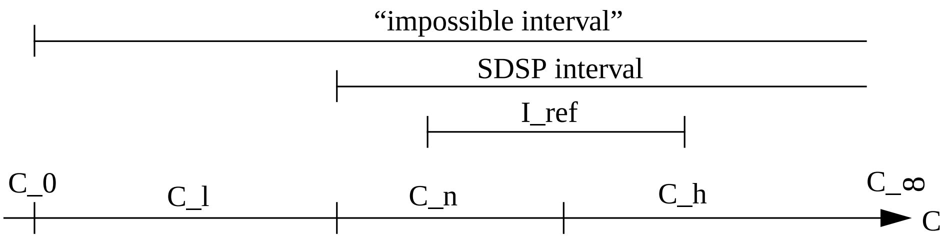

Figure 7: Sketch of the proof of soundness of the SDSP method

If we neglected these rules or ignored their on-step evaluation, the SDSP method would become unsound. This can be easily seen by investigating the example in Figure 6. If the current divider rule had been omitted, the qualitative values describing I8 would have to be determined by evaluating Ohm's law ($I8 = U8 \cdot \frac{1}{S8}$) only. Applying Ohm‘s law only, the qualitative values I8_l or I8_n or I8_h are obtained and, therefore, the SDSP is no longer sound. If the current divider rule is evaluated by applying elementary qualitative operators, again, the qualitative values I8_l or I8_n or I8_h are obtained.

Furthermore, the soundness of the SDSP method is achieved by the special forms of the electrical rules, that are specifically chosen in order to optimize the derivation of their qualitative versions. This can be demonstrated by applying elementary qualitative operators to $P7 = \frac{R23 \cdot R5}{R23 + R5}$ (see also equation (16)). As a result, P7_l or P7_n or P7_h were obtained, and, therefore, the SDSP method would be unsound. For a more detailed presentation of the proof we refer to [Milde, 1997].

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