Definition of Image Understanding

Image understanding is the task-oriented reconstruction and interpretation of a scene by means of images

scene:	section of the real world stationary (3D) or moving (4D)
image:	view of a scene projection, density image (2D) depth image (2 1/2D) image sequence (3D)
reconstruction and interpretation:	computer-internal scene description quantitative + qualitative + symbolic
task-oriented:	for a purpose, to fulfill a particular task context-dependent, supporting actions of an agent

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Illustration of Image Understanding

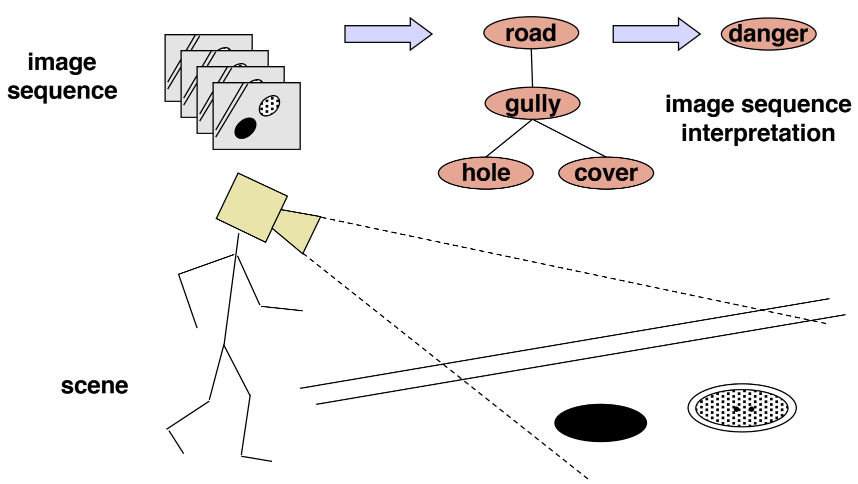

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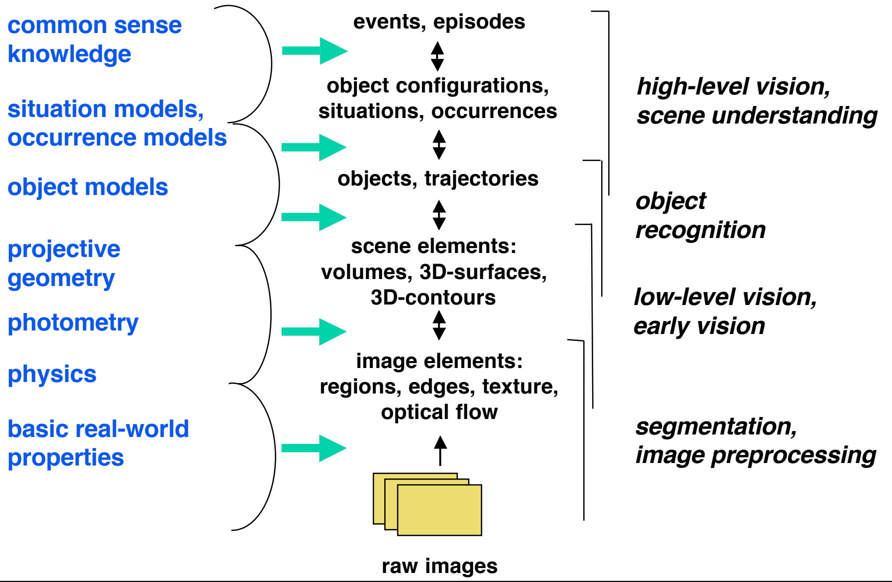
Image Understanding as a Knowledge-based Process



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Abstraction Levels for the Description of Computer Vision Systems

Knowledge level

What knowledge or information enters a process? What knowledge or information is obtained by a process?

What are the laws and constraints which determine the behavior of a process?

Algorithmic level

How is the relevant information represented?

What algorithms are used to process the information?

Implementation level

What programming language is used?

What computer hardware is used?

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Example for Knowledge-level Analysis

What knowledge or information enters a process? What knowledge or information is obtained by a process?

What are the laws and constraints which determine the behavior of a process?

Consider shape-from-shading:

In order to obtain the 3D shape of an object, it is necessary to
- **state what knowledge is available (greyvalues, surface properties, illumination direction, etc.)**
- **state what information is desired (e.g. qualitative vs. quantitative)**
- **exploit knowledge about the physics of image formation**

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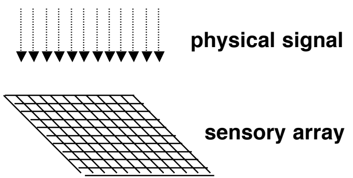
Image Formation

Images can be generated by various processes:

- **illumination of surfaces, measurement of reflections** ← "natural images"
- **illumination of translucent material, measurement of irradiation**
- **measurement of heat (infrared) radiation**
- **X-ray of material, computation of density map**
- **measurement of any features by means of a sensory array**



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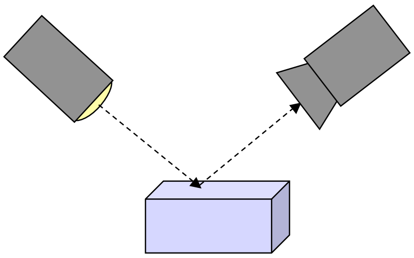
Formation of Natural Images



Intensity (brightness) of a pixel depends on

1. illumination (spectral energy, secondary illumination)
2. object surface properties (reflectivity)
3. sensor properties
4. geometry of light-source, object and sensor constellation (angles, distances)
5. transparency of irradiated medium (mistiness, dustiness)

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Qualitative Surface Properties

When light hits a surface, it may be

- absorbed
- reflected
- scattered

in general, all effects may be mixed

Simplifying assumptions:

- Radiance leaving at a point is due to radiance arriving at this point
- All light leaving the surface at a wavelength is due to light arriving at the same wavelength
- Surface does not generate light internally

The "amount" of reflected light may depend on:

- the "amount" of incoming light
- the angles of the incoming light w.r.t. to the surface orientation
- the angles of the outgoing light w.r.t. to the surface orientation

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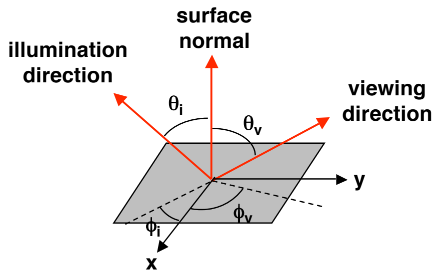
Photometric Surface Properties



θ_i, θ_v polar (zenith) angles

ϕ_i, ϕ_v azimuth angles

In general, the ability of a surface to reflect light is given by the Bi-directional Reflectance Distribution Function (BRDF) r:

$$r(\theta_i, \phi_i; \theta_v, \phi_v) = \frac{\delta L(\theta_v, \phi_v)}{\delta E(\theta_i, \phi_i)}$$

δE = irradiance of light source received by the surface patch

δL = radiance of surface patch towards viewer

For many materials the reflectance properties are rotation invariant, in this case the BRDF depends on θ_i, θ_v, ϕ, where $\phi = \phi_i - \phi_v$.

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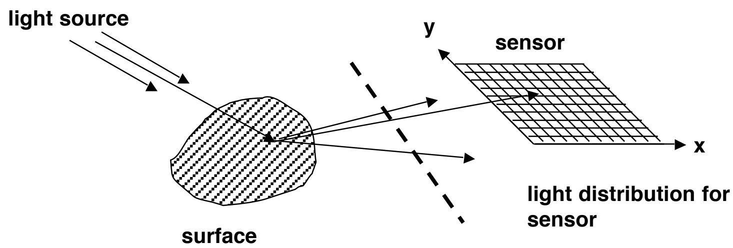
Intensity of Sensor Signals



Intensities of sensor signals depend on
- location x, y on sensor plane
- instance of time t
- frequency of incoming light wave λ
- spectral sensitivity of sensor

$$f(x,y,t) = \int_0^\infty C(x,y,t,\lambda)\, S(\lambda)\, d\lambda$$

sensitivity function of sensor

spectral energy distribution

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Multispectral Images

Sensors with separate channels of different spectral sensitivities generate multispectral images:

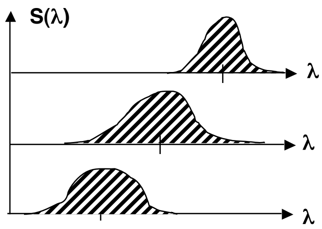
$$f_1(x,y,t) = \int_0^\infty C(x,y,t,\lambda)S_1(\lambda)\,d\lambda$$

$$f_2(x,y,t) = \int_0^\infty C(x,y,t,\lambda)S_2(\lambda)\,d\lambda$$

$$f_3(x,y,t) = \int_0^\infty C(x,y,t,\lambda)S_3(\lambda)\,d\lambda$$

Example:

R (red)	650 nm center frequency
G (green)	530 nm center frequency
B (blue)	410 nm center frequency



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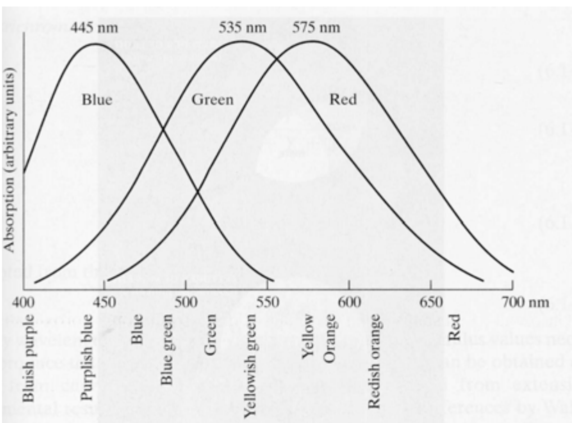
Spectral Sensitivity of Human Eyes



Standardized wavelengths:
red = 700 nm, green = 546.1 nm, blue = 435.8 nm

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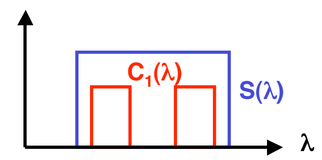
Non-unique Sensor Response

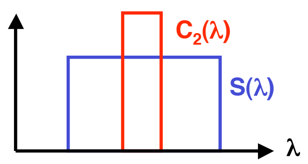
Different spectral distributions may lead to identical sensor responses and hence cannot be distinguished

$$f(x,y,t) = \int_0^\infty C_1(x,y,t,\lambda)S(\lambda)d\lambda = \int_0^\infty C_2(x,y,t,\lambda)S(\lambda)d\lambda$$

different spectral energy distributions

<u>Example</u>:





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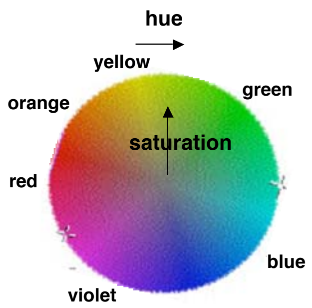
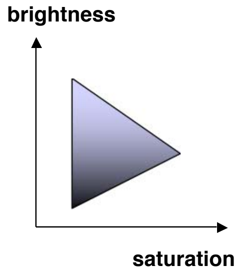
Dimensions of Colour

Human perception of colour distinguishes between 3 dimensions:
- hue
- saturation
- brightness

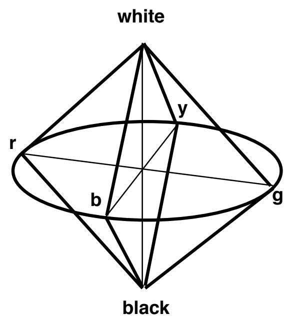
(hue and saturation: chromaticity)



colour circle



NCS* colour spindle



* Swedish Natural Colour System

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RGB Images of a Natural Scene

Here, single colour images are rendered as greyvalue intensity images:
stronger spectral intensity = more brightness

R+G+B	R	G	B
			

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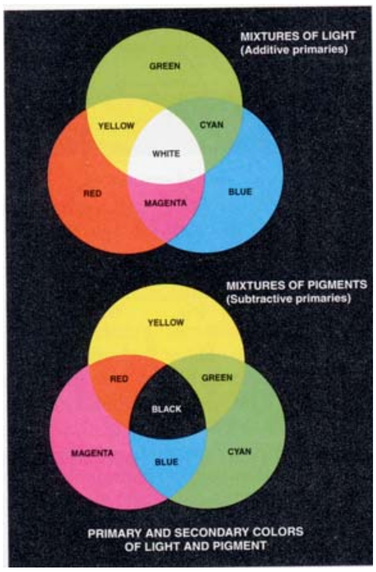
Primary and Secondary Colours



Primary colours:

red, green, blue

Secondary colours:

magenta = red + blue
cyan = green + blue
yellow = red + green

aus: Gonzales & Woods
Digital Image Processing
Prentice Hall 2002

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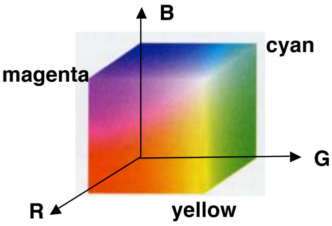
Technical Colour Models

RGB colour model



Typical discretization:

8 bits per colour dimension
=> 16,77,216 colours

CMY colour model

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

HSI colour model

Hue:

$$H = \begin{cases} \Theta & \text{if } B \leq G \\ 360 - \Theta & \text{if } B > G \end{cases}$$

$$\Theta = \cos^{-1} \frac{1/2\,[(R-G) + (R-B)]}{[(R-G)^2 + (R-B)(G-B)]^{1/2}}$$

Saturation:

$$S = 1 - \frac{3}{(R + G + B)} [\min(R, G, B)]$$

Intensity:

$$I = 1/3\,(R + G + B)$$

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Discretization of Images

Image functions must be discretized for computer processing:

- spatial quantization
  the image plane is represented by a 2D array of picture cells
- greyvalue quantization
  each greyvalue is taken from a discrete value range
- temporal quantization
  greyvalues are taken at discrete time intervals

$$\begin{aligned} f(x,y,t) \Rightarrow \{\, & f_s(x_1, y_1,t_1), f_s(x_2, y_2,t_1), f_s(x_3, y_3,t_1), \ldots \\ & f_s(x_1, y_1,t_2), f_s(x_2, y_2,t_2), f_s(x_3, y_3,t_2), \ldots \\ & f_s(x_1, y_1,t_3), f_s(x_2, y_2,t_3), f_s(x_3, y_3,t_3), \ldots \,\} \end{aligned}$$

A single value of the discretized image function is called a pixel (picture element).

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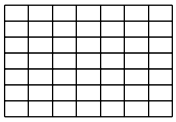
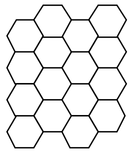
Spatial Quantization

Rectangular grid

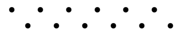


Greyvalues represent the quantized value of the signal power falling into a grid cell.

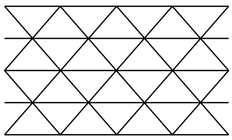
Hexagonal grid



Note that samples of a hexagonal grid are equally spaced along rows, with successive rows shifted by half a sampling interval.



Triangular grid



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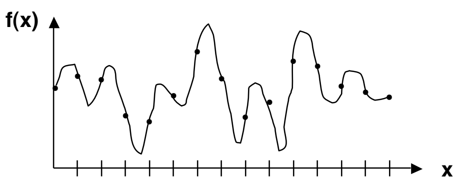
Reconstruction from Samples

Under what conditions can the original (continuous) signal be reconstructed from its sampled version?

Consider a 1-dimensional function f(x):



Reconstruction is only possible, if "variability" of function is restricted.

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Sampling Theorem

Shannon´s Sampling Theorem:

A bandlimited function with bandwidth W can be exactly reconstructed from equally spaced samples, if the sampling distance is not larger than $\frac{1}{2W}$

bandwidth = largest frequency contained in signal

(=> Fourier decomposition of a signal)

Analogous theorem holds for 2D signals with limited spatial frequencies W_x and W_y

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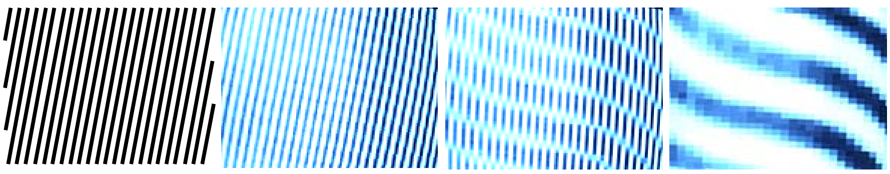
Aliasing

Sampling an image with fewer samples than required by the sampling theorem may cause "aliasing" (artificial structures).

Example:



original | **143 x 128** | **71 x 64** | **35 x 32**

To avoid aliasing, bandwidth of image must by reduced prior to sampling.

(=> low-pass filtering)

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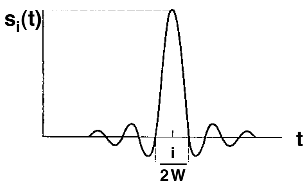
Reconstructing the Image Function from Samples

Formally, a continuous function f(t) with bandwidth W can be exactly reconstructed using <u>sampling functions</u> $s_i(t)$:

$$s_i(t) = \sqrt{2W}\,\frac{\sin 2\pi W[t - i/(2W)]}{2\pi W[t - i/(2W)]}$$

$$x(t) = \sum_{i=-\infty}^{\infty} \sqrt{\frac{1}{2W}}\; x\,\underbrace{\left(\frac{i}{2W}\right)}_{\text{sample values}}\, S_i(t)$$

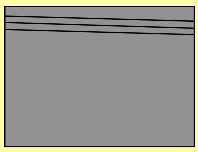


An analogous equation holds for 2D.

In practice, image functions are generated from samples by interpolation.

Sampling TV Signals

PAL standard:
- picture format 3 : 4
- 25 full frames (50 half frames) per second
- interlaced rows: 1, 3, 5, ... , 2, 4, 6, ...
- 625 rows per full frame, 576 visible
- 64 µs per row, 52 µs visible
- 5 MHz bandwidth



Only 1D sampling is required because of fixed row structure.

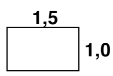
Sampling intervals of $\Delta t = 1/(2W) = 10^{-7}s = 100$ ns give maximal possible resolution.

With $\Delta t = 100$ ns, a row of duration 52 µs gives rise to 520 samples.

In practice, one often chooses <u>512 pixels per TV row</u>.

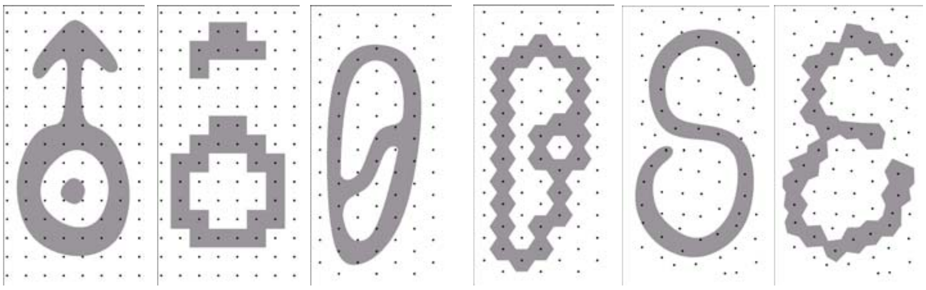
=> 576 x 512 = 294912 pixels per full frame

=> rectangular pixel size with width/height $= \left(\frac{4}{512}\right) / \left(\frac{3}{576}\right) = 1{,}5$



Sampling of Binary Images (1)

Problem: Shapes may change under digitization



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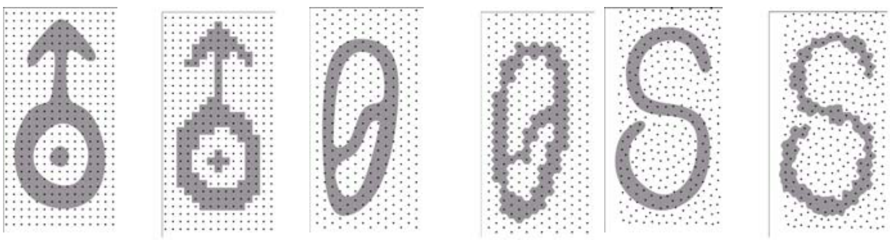
Sampling of Binary Images (2)

Problem: Shapes may change under digitization



This cannot be solved by using Shannon´s Theorem since binary images are not bandlimited.

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Shape Preserving Sampling Theorem (1)

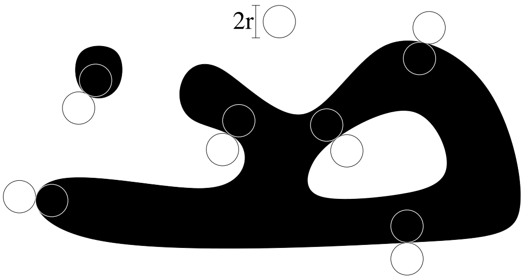
Shape Preserving Sampling Theorem:
The shape of an r-regular image can be correctly reconstructed after sampling with any sampling grid, if the grid point distance is not larger than r.
Stelldinger, Köthe 2003

grid point distance: maximal distance from arbitrary point to the next sampling point

r-regular binary image:

osculating r-discs at each boundary point of the shape

⇒ curvature bounded by 1/r
⇒ bounded thinness of parts
⇒ minimal distance between parts



Shape Preserving Sampling Theorem (2)

Shape Preserving Sampling Theorem:
The shape of an r-regular image can be correctly reconstructed after sampling with any sampling grid, if the grid point distance is not larger than r.
Stelldinger, Köthe 2003

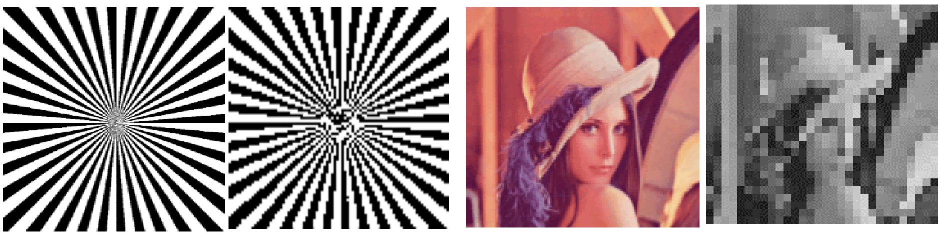
What does correct reconstruction mean?

Topological and geometric similarity criterion:
One shape can be mapped onto the other by twisting the whole plane, such that the displacement of each point is smaller than r.

Sampling of Shapes in Arbitrary Images (1)

The previous sampling theorem also holds for greyvalue images, if each level set is an r-regular shape.

A level set is the set where the image is brighter than a given threshold value.



sampling + reconstruction

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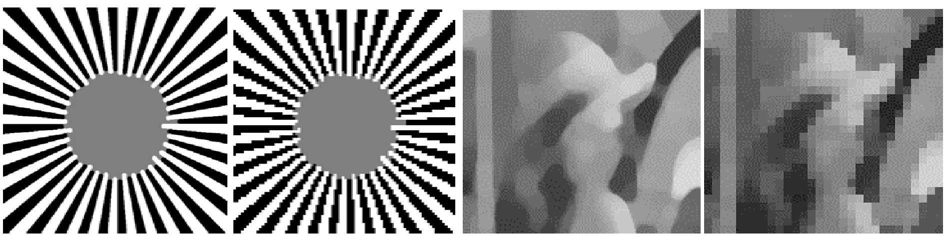
Sampling of Shapes in Arbitrary Images (2)

Reconstruction after sampling from r-regular originals



The generalization to higher dimensions is still an unsolved problem!

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Comparison of the Sampling Theorems

	Shannon´s Sampling Theorem	Shape Preserving Sampling Theorem
necessary image property	bandlimited with bandwidth W	r-regular
equation	$\left(\frac{r'}{\sqrt{2}} = \right) \; d < \frac{1}{2W}$	$r' < r$
reconstructed image	identical to original image	same shape as the original image
prefiltering	band-limitation: efficient algorithms (but shapes may change!)	regularization: unsolved problem
2D sampling grid	rectangular grid	arbitrary grids
dimension of definition	1D (generalizable to n-D)	2D (partly generalizable to n-D)

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Quantization of Greyvalues

Quantization of greyvalues transforms continuous values of a sampled image function into digital quantities.

Typically 2 ... 2^{10} quantization levels are used, depending on task.

Less than 2^9 quantization levels may cause artificial contours for human perception.

<u>Example</u>:

256 16 8 4 2

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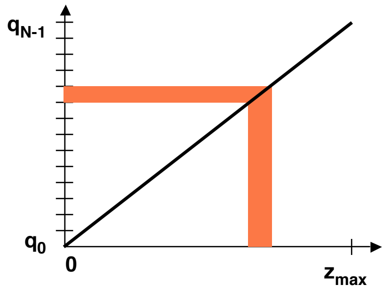
Uniform Quantization

Equally spaced discrete values q_0 ... q_{N-1} represent equal-width greyvalue intervals of the continuous signal.

Typically $N = 2^K$ for $K = 1 ... 10$



Uniform quantization may "waste" quantization levels, if greyvalues are not equally distributed.

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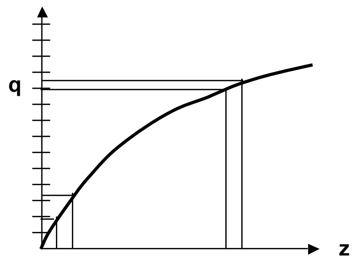
Nonlinear Quantization Curves

E.g. fine resolution for "interesting" greyvalue ranges, coarse resolution for less interesting greyvalue ranges.

<u>Example</u>:
Low greyvalues are mapped into more quantization levels than high greyvalues.



<u>Note</u>:

Subjective brightness in human perception depends (roughly) logarithmically on the actual (measurable) brightness.

To let the computer see brightness as humans, use a logarithmic quantization curve.

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Optimal Quantization (1)

<u>Assumption</u>:
Probability density p(z) for continuous greyvalues and number of quantization levels N are known.

<u>Goal</u>:
Minimize mean quadratic quantization error d_q by choosing optimal interval boundaries z_n and optimal discrete representatives q_n.

$$d_q^2 = \sum_{n=0}^{N-1} \int_{z_n}^{z_{n+1}} (z - q_n)^2 p(z) dz$$

Minimizing by setting the derivatives zero:

$$\frac{\delta}{\delta z_n} d_q^2 = (z_n - q_{n-1})^2 p(z_n) - (z_n - q_n)^2 p(z_n) = 0 \quad \text{for } n = 1 \ldots N-1$$

$$\frac{\delta}{\delta q_n} d_q^2 = -2 \int_{z_n}^{z_{n+1}} (z - q_n) p(z) dz = 0 \quad \text{for } n = 0 \ldots N-1$$

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Optimal Quantization (2)

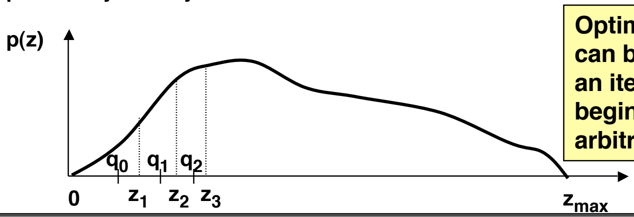
Solution for optimal quantization:

$$z_n = \frac{1}{2}(q_{n-1} + q_n) \quad \text{for } n = 1 \ldots N-1 \text{ when } p(z_n) > 0$$

Each interval boundary must be in the middle of the corresponding quantization levels.

$$q_n = \frac{\int_{z_n}^{z_{n+1}} z p(z) dz}{\int_{z_n}^{z_{n+1}} p(z) dz} \quad \text{for } n = 0 \ldots N-1$$

Each quantization level is the center-of-gravity coordinate of the corresponding probability density area.



Optimal quantization can be determined by an iterative algorithm beginning with an arbitrary choice of z_1

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Binarization

For many applications it is convenient to distinguish only between 2 greyvalues, "black" and "white", or "1" and "0".

Example: Separate object from background

Binarization = transforming an image function into a binary image

Thresholding:

$$g(x, y) \Rightarrow \begin{cases} 0 & \text{if } g(x, y) < T \\ 1 & \text{if } g(x, y) \geq T \end{cases} \qquad T \text{ is threshold}$$

Thresholding is often applied to digital images in order to isolate parts of the image, e.g. edge areas.

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Threshold Selection by Trial and Error

A threshold which perfectly isolates an image component must not always exist.

Selection by trial and error:

Select threshold until some image property is fulfilled, e.g.

$$q = \frac{\#\ \text{white pixels}}{\#\ \text{black pixels}} \Rightarrow q_0$$

line strength $\Rightarrow d_0$

number of connected components $\Rightarrow n_0$

Number of trials may be small if logarithmic search can be used.

Example:

At most 8 trials are needed to select a threshold $0 \leq T \leq 255$ which best approximates a given q_0.

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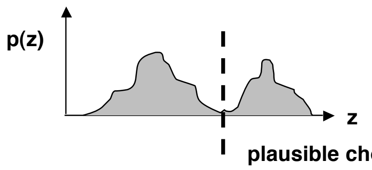
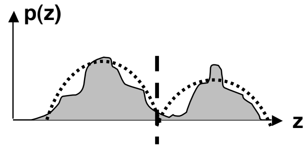
Distribution-based Threshold Selection

The greyvalue distribution of the image function may show a bimodality:



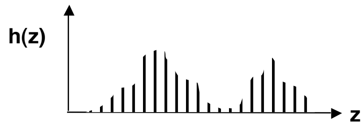
Two methods for finding a plausible threshold:



1. Find "valley" between two "hills"

2. Fit hill templates and compute intersection

Typically, computations are based on histograms which provide a discrete approximation of a distribution.



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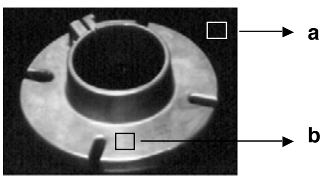
Threshold Selection Based on Reference Positions

In many applicatons, the approximate position of image components is known a priori. These positions may provide useful reference greyvalues.

Example:



possible choice of threshold:

$$T = \frac{a + b}{2}$$

Threshold selection and binarization may be decisively facilitated by a good choice of illumination and image capturing techniques.

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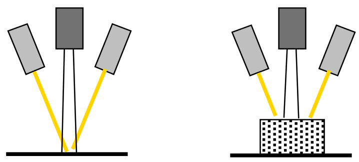
Image Capturing for Thresholding

If the image capturing process can be controlled, thresholding can be facilitated by a suitable choice of

- illumination
- camera position
- object placement
- background greyvalue or colour
- preprocessing

Example: Backlighting
Illumination from the rear gives bright background and shadowed object

Example: Slit illumination
On a conveyor belt illuminated by a light slit at an angle, elevations give rise to displacements which can be recognized by a camera.



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Perspective Projection Transformation

Where does a point of a scene appear in an image?

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{?} \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$

Transformation in 3 steps:

1. scene coordinates => camera coordinates
2. projection of camera coordinates into image plane
3. camera coordinates => image coordinates

Perspective projection equations are essential for Computer Graphics. For Image Understanding we will need the inverse: What are possible scene coordinates of a point visible in the image? This will follow later.

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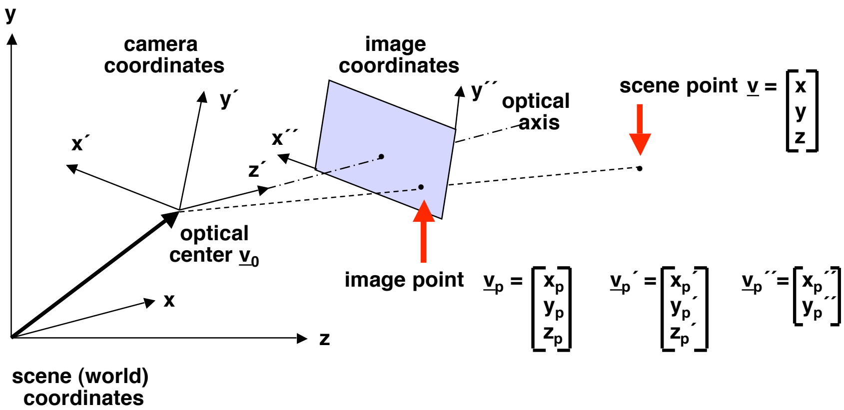
Perspective Projection in Independent Coordinate Systems

It is often useful to describe real-world points, camera geometry and image points in separate coordinate systems. The formal description of projection involves transformations between these coordinate systems.



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3D Coordinate Transformation (1)

The new coordinate system is specified by a translation and rotation with respect to the old coordinate system:

$\underline{v}' = R(\underline{v} - \underline{v}_0)$

$\underline{v}_0$ is displacement vector
R is rotation matrix

R may be decomposed into 3 rotations about the coordinate axes:

$R = R_x R_y R_z$

α = rotation angle about x-axis
β = rotation angle about y-axis
γ = rotation angle about z-axis

("nick angle", "pan angle", and "tilt angle" for the camera coordinate assignment shown before)

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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3D Coordinate Transformation (2)

By multiplying the 3 matrices R_x, R_y and R_z, one gets

$$R = \begin{bmatrix} \cos\beta\cos\gamma & \cos\beta\sin\gamma & \sin\beta \\ -\sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & -\sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & -\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\cos\beta \end{bmatrix}$$

For formula manipulations, one tries to avoid the trigonometric functions and takes

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Note that the coefficients of R are constrained:
A rotation matrix is orthonormal:

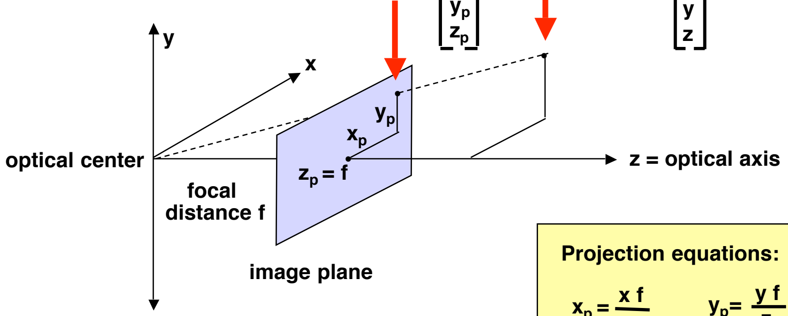
$R\,R^T = I$ (unit matrix)

Perspective Projection Geometry

Projective geometry relates the coordinates of a point in a scene to the coordinates of its projection onto an image plane.

Perspective projection is an adequate model for most cameras.

image point $\underline{V}_p = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix}$ scene point $\underline{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$



Projection equations:

$$x_p = \frac{x\,f}{z} \qquad y_p = \frac{y\,f}{z}$$

Perspective and Orthographic Projection

Within the camera coordinate system the perspective projection of a scene point onto the image plane is described by

$$x_p' = \frac{x'f}{z'} \qquad y_p' = \frac{y'f}{z'} \qquad z_p' = f \qquad (f = \text{focal distance})$$

- nonlinear transformation
- loss of information

If all objects are far away (large z´), f/z´ is approximately constant

=> orthographic projection

$$x_p' = s\,x' \quad y_p' = s\,y' \qquad (s = \text{scaling factor})$$

Orthographic projection can be viewed as projection with parallel rays + scaling

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From Camera Coordinates to Image Coordinates

Transform may be necessary because
- optical axis may not penetrate image plane at origin of desired coordinate system
- transition to discrete coordinates may require scaling

$x_p'' = (x_p' - x_{p0}')\,a$ — a, b scaling parameters

$y_p'' = (y_p' - y_{p0}')\,b$ — x_{p0}', y_{p0}' origin of image coordinate system

Example:

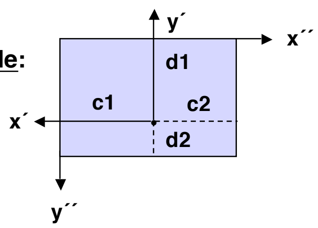

Image boundaries in camera coordinates:
$x'_{max} = c1 \quad x'_{min} = c2$
$y'_{max} = d1 \quad y'_{min} = d2$

Discrete image coordinates:
$x'' = 0 .. 511 \quad y'' = 0 .. 575$

Transformation parameters:
$x_{p0}' = c1 \quad y_{p0}' = d1 \quad a = 512 / (c2 - c1) \quad b = 576 / (d2 - d1)$

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Complete Perspective Projection Equation

We combine the 3 transformation steps:

1. scene coordinates => camera coordinates
2. projection of camera coordinates into image plane
3. camera coordinates => image coordinates

$x_p'' = \{ f/z' [\cos\beta \cos\gamma \ (x - x_0) + \cos\beta \sin\gamma \ (y - y_0) + \sin\beta \ (z - z_0)] - x_{p0} \}\ a$

$y_p'' = \{ f/z' [(- \sin\alpha \sin\beta \cos\gamma - \cos\alpha \sin\gamma) \ (x - x_0) +$
$(- \sin\alpha \sin\beta \sin\gamma + \cos\alpha \cos\gamma) \ (y - y_0) +$
$\sin\alpha \cos\beta \ (z - z_0)] - y_{p0} \}\ b$

with $z' = (- \cos\alpha \sin\beta \cos\gamma + \sin\alpha \sin\gamma) \ (x - x_0) +$
$(- \cos\alpha \sin\beta \sin\gamma - \sin\alpha \cos\gamma) \ (y - y_0) +$
$\cos\alpha \cos\beta \ (z - z_0)$

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Homogeneous Coordinates (1)

4D notation for 3D coordinates which allows to express nonlinear 3D transformations as linear 4D transformations.

Normal: $\underline{v}' = R\ (\underline{v} - \underline{v}_0)$

Homogeneous coordinates: $\underline{\mathit{v}}' = \mathit{A}\ \underline{\mathit{v}}$ *(note italics for homogeneous coordinates)*

$$\mathit{A} = \mathit{R}\ \mathit{T} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transition to homogeneous coordinates:

$\underline{v}^T = [x\ y\ z]$ => $\underline{\mathit{v}}^T = [wx\ wy\ wz\ w]$ $w \neq 0$ is arbitrary constant

Return to normal coordinates:

1. Divide components 1- 3 by 4th component
2. Omit 4th component

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Homogeneous Coordinates (2)

Perspective projection in homogeneous coordinates:

$$\underline{v}_p{}' = P\,\underline{v}' \quad \text{with } P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \quad \text{and } \underline{v}' = \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix} \quad \text{gives } \underline{v}_p{}' = \begin{bmatrix} wx \\ wy \\ wz \\ wz/f \end{bmatrix}$$

Returning to normal coordinates gives $\underline{v}_p{}' = \begin{bmatrix} xf/z \\ yf/z \\ f \end{bmatrix}$

compare with earlier slide

Transformation from camera into image coordinates:

$$\underline{v}_p{}'' = B\,\underline{v}_p{}' \quad \text{with } B = \begin{bmatrix} a & 0 & 0 & -x_0a \\ 0 & b & 0 & -y_0b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and } \underline{v}_p{}' = \begin{bmatrix} wx_p \\ wy_p \\ 0 \\ w \end{bmatrix} \quad \text{gives } \underline{v}_p{}'' = \begin{bmatrix} wa(x_p-x_0) \\ wb(y_p-y_0) \\ 0 \\ w \end{bmatrix}$$

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Homogeneous Coordinates (3)

Perspective projection can be completely described in terms of a linear transformation in homogeneous coordinates:

$$\underline{v}_p{}'' = B\,P\,R\,T\,\underline{v}$$

B P R T may be combined into a single 4 x 4 matrix *C* :

$$\underline{v}_p{}'' = C\,\underline{v}$$

In the literature the parameters of these equations may vary because of different choices of coordinate systems, different order of translation and rotation, different camera models, etc.

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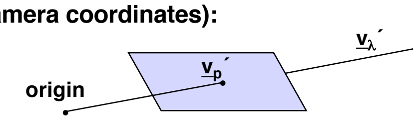
Inverse Perspective Equations

Which points in a scene correspond to a point in the image?

$$\begin{bmatrix} x_p'' \\ y_p'' \end{bmatrix} \xrightarrow{?} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Each image point defines a projection ray as the locus of possible scene points (for simplicity in camera coordinates):

$\underline{v}_p' \Rightarrow \underline{v}_\lambda' = \lambda\, \underline{v}_p'$

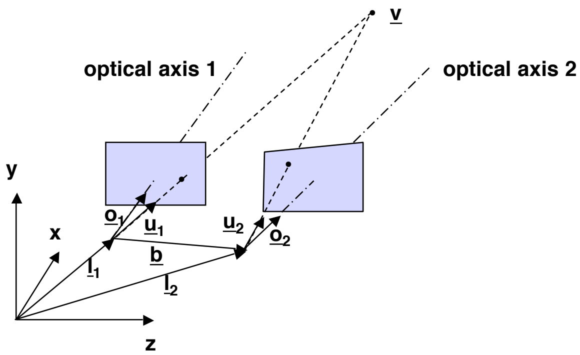


$$\boxed{\underline{v} = \underline{v}_0 + R^T \lambda\, \underline{v}_p'}$$

3 equations with the 4 unknowns x, y, z, λ and camera parameters R and $\underline{v}_0$

Applications of inverse perspective mapping for e.g.
- distance measurements
- binocular stereo
- camera calibration
- motion stereo

Binocular Stereo (1)



$\underline{l}_1$, $\underline{l}_2$	camera positions (optical center)
$\underline{b}$	stereo base
$\underline{o}_1$, $\underline{o}_2$	camera orientations (unit vectors)
f_1, f_2	focal distances
$\underline{v}$	scene point
$\underline{u}_1$, $\underline{u}_2$	projection rays of scene point (unit vectors)

Binocular Stereo (2)

Determine distance to $\underline{v}$ by measuring $\underline{u}_1$ and $\underline{u}_2$

Formally: $\alpha\, \underline{u}_1 = \underline{b} + \beta\, \underline{u}_2 \quad \Rightarrow \quad \underline{v} = \alpha\, \underline{u}_1 + \underline{l}_1$

α and β are overconstrained by the vector equation. In practice, measurements are inexact, no exact solution exists (rays do not intersect).

Better approach: Solve for the point of closest approximation of both rays:

$$\underline{v} = \frac{\alpha_0\, \underline{u}_1 + (\underline{b} + \beta_0\, \underline{u}_2)}{2} + \underline{l}_1 \qquad \Rightarrow \qquad \text{minimize} \quad \| \alpha\, \underline{u}_1 - (\underline{b} + \beta\, \underline{u}_2) \|^2$$

Solution:

$$\alpha_0 = \frac{\underline{u}_1^T \underline{b} - (\underline{u}_1^T \underline{u}_2)\,(\underline{u}_2^T \underline{b})}{1 - (\underline{u}_1^T \underline{u}_2)^2}$$

$$\beta_0 = \frac{(\underline{u}_1^T \underline{u}_2)\,(\underline{u}_1^T \underline{b}) - (\underline{u}_2^T \underline{b})}{1 - (\underline{u}_1^T \underline{u}_2)^2}$$

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Distance in Digital Images

Intuitive concepts of continuous images do not always carry over to digital images.

Several methods for measuring distance between pixels:

Eucledian distance

$D_E((i,j),(h,k)) = \sqrt{(i-h)^2 + (j-k)^2}$

costly computation of square root, can be avoided for distance comparisons

City block distance

$D_4((i, j)(h, k)) = |i - h| + |j - k|$

number of horizontal and vertical steps in a rectangular grid

Chessboard distance

$D_8((i, j)(h, k)) = \max \{ |i - h|, |j - k| \}$

number of steps in a rectangular grid if diagonal steps are allowed (number of moves of a king on a chessboard)

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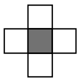
Connectivity in Digital Images

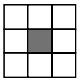
Connectivity is an important property of subsets of pixels. It is based on adjacency (or neighbourhood):

Pixels are 4-neighbours if their distance is $D_4 = 1$



all 4-neighbours of center pixel

Pixels are 8-neighbours if their distance is $D_8 = 1$



all 8-neighbours of center pixel

A path from pixel P to pixel Q is a sequence of pixels beginning at Q and ending at P, where consecutive pixels are neighbours.

In a set of pixels, two pixels P and Q are connected, if there is a path between P and Q with pixels belonging to the set.

A region is a set of pixels where each pair of pixels is connected.

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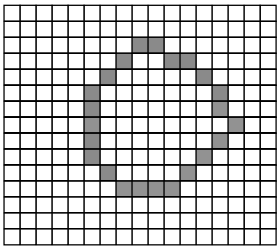
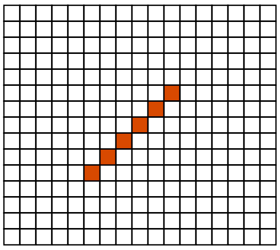
Closed Curve Paradoxon



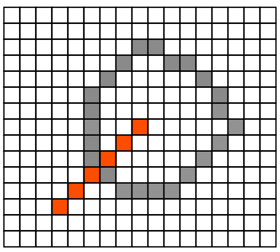
line 1



line 2

solid lines if 8-neighbourhood is used

a similar paradoxon arises if 4-neighbourhoods are used



line 2 does not intersect line 1 although it crosses from the outside to the inside

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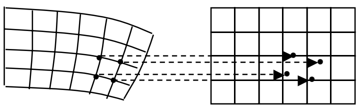
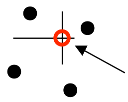
Geometric Transformations

Various applications:

- change of view point
- elimination of geometric distortions from image capturing
- registration of corresponding images
- artificial distortions, Computer Graphics applications

Step 1: Determine mapping $\underline{T}(x, y)$ from old to new coordinate system
Step 2: Compute new coordinates (x', y') for (x, y)
Step 3: Interpolate greyvalues at grid positions from greyvalues at transformed positions





Polynomial Coordinate Transformations

General format of transformation:

$$x' = \sum_{r=0}^{m} \sum_{k=0}^{m-r} a_{rk} x^r y^k$$

$$y' = \sum_{r=0}^{m} \sum_{k=0}^{m-r} b_{rk} x^r y^k$$

- Assume polynomial mapping between (x, y) and (x', y') of degree m
- Determine corresponding points
- a) Solve linear equations for a_{rk}, b_{rk} $(r, k = 1 \ldots m)$
  b) Minimize mean square error (MSE) for point correspondences

Approximation by biquadratic transformation:

$x' = a_{00} + a_{10}x + a_{01}y + a_{11}xy + a_{20}x^2 + a_{02}y^2$
$y' = b_{00} + b_{10}x + b_{01}y + b_{11}xy + b_{20}x^2 + b_{02}y^2$

at least 6 corresponding pairs needed

Approximation by affine transformation:

$x' = a_{00} + a_{10}x + a_{01}y$
$y' = b_{00} + b_{10}x + b_{01}y$

at least 3 corresponding pairs needed

Translation, Rotation, Scaling, Skewing

Translation by vector $\underline{t}$:

$\underline{v}' = \underline{v} + \underline{t}$ with $\underline{v}' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $\underline{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ $\underline{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$

Rotation of image coordinates by angle α:

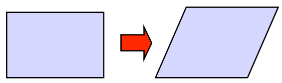
$\underline{v}' = R\,\underline{v}$ with $R = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Scaling by factor a in x-direction and factor b in y-direction:

$\underline{v}' = S\,\underline{v}$ with $S = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Skewing by angle β:

$\underline{v}' = W\,\underline{v}$ with $W = \begin{bmatrix} 1 & \tan\beta \\ 0 & 1 \end{bmatrix}$



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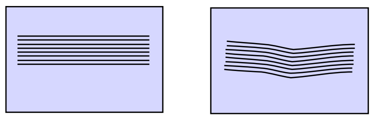
Example of Geometry Correction by Scaling

Distortions of electron-tube cameras may be

1 - 2 % => more than 5 lines for TV images



Correction procedure may be based on

- fiducial marks engraved into optical system
- a test image with regularly spaced marks

Ideal mark positions:

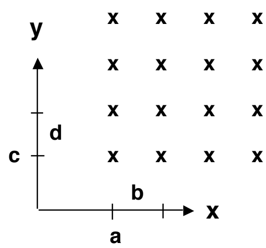
$x_{mn} = a + mb$, $y_{mn} = c + nd$

$m = 0 \ldots M-1$

$n = 0 \ldots N-1$

Actual mark positions:

x'_{mn}, y'_{mn}



Determine a, b, c, d such that MSE (mean square error) of deviations is minimized

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Minimizing the MSE

Minimize $$E = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (x_{mn} - x'_{mn})^2 + (y_{mn} - y'_{mn})^2$$

$$= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (a + mb - x'_{mn})^2 + (c + nd - y'_{mn})^2$$

From $\delta E/\delta a = \delta E/\delta b = \delta E/\delta c = \delta E/\delta d = 0$ we get:

$$a = \frac{2}{MN(M+1)} \sum_m \sum_n (2M - 1 - 3m)\ x'_{mn}$$

$$b = \frac{6}{MN(M^2 - 1)} \sum_m \sum_n (2m - M + 1) x'_{mn}$$

$$c = \frac{2}{MN(N+1)} \sum_m \sum_n (2N - 1 - 3n)\ y'_{mn}$$

$$d = \frac{6}{MN(N^2 - 1)} \sum_m \sum_n (2n - N + 1) y'_{mn}$$

Special case M=N=2:

$a = 1/2\ (x'_{00} + x'_{01})$

$b = 1/2\ (x'_{10} - x'_{00} + x'_{11} - x'_{01})$

$c = 1/2\ (y'_{00} + y'_{01})$

$d = 1/2\ (y'_{01} - y'_{00} + y'_{11} - y'_{10})$

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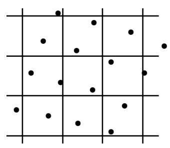
Principle of Greyvalue Interpolation

Greyvalue interpolation = computation of unknown greyvalues at locations (u´v´) from known greyvalues at locations (x´y´)



Two ways of viewing interpolation in the context of geometric transformations:

A Greyvalues at grid locations (x y) in old image are placed at corresponding locations (x´y´) in new image: g(x´y´) = g(T(x y))

=> interpolation in new image

B Grid locations (u´v´) in new image are transformed into corresponding locations (u v) in old image: g(u v) = g(T^{-1}(u´v´))

=> interpolation in old image

We will take view B:

Compute greyvalues between grid from greyvalues at grid locations.

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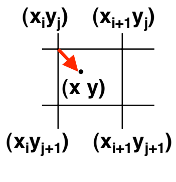
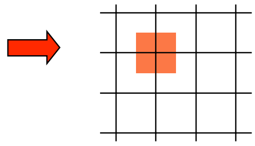
Nearest Neighbour Greyvalue Interpolation

Assign to (x y) greyvalue of nearest grid location

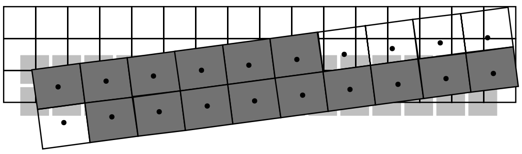


$(x_i y_j)$ $(x_{i+1} y_j)$ $(x_i y_{j+1})$ $(x_{i+1} y_{j+1})$	grid locations
(x y)	location between grid with $x_i \le x \le x_{i+1}$, $y_j \le y \le y_{j+1}$

Each grid location represents the greyvalues in a rectangle centered around this location:



Straight lines or edges may appear step-like after this transformation:



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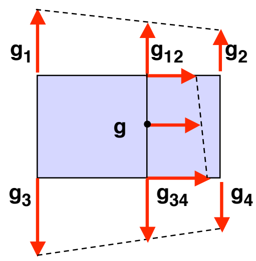
Bilinear Greyvalue Interpolation

The greyvalue at location (x y) between 4 grid points $(x_i y_j)$ $(x_{i+1} y_j)$ $(x_i y_{j+1})$ $(x_{i+1} y_{j+1})$ is computed by linear interpolation in both directions:

$$g(x,y) = \frac{1}{(x_{i+1}-x_i)(y_{j+1}-y_j)} \Big\{ (x_{i+1}-x)(y_{j+1}-y) g(x_i y_j) + (x-x_i)(y_{j+1}-y) g(x_{i+1} y_j) + (x_{i+1}-x)(y-y_j) g(x_i y_{j+1}) + (x-x_i)(y-y_j) g(x_{i+1} y_{j+1}) \Big\}$$

Simple idea behind long formula:

1. Compute g_{12} = linear interpolation of g_1 and g_2
2. Compute g_{34} = linear interpolation of g_3 and g_4
3. Compute g = linear interpolation of g_{12} and g_{34}



The step-like boundary effect is reduced.
But bilear interpolation may blur sharp edges.

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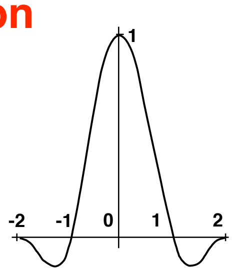
Bicubic Interpolation

Each greyvalue at a grid point is taken to represent the center value of a local bicubic interpolation surface with cross section h_3.

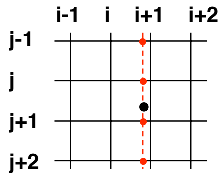
$$h_3 = \begin{cases} 1 - 2|x|^2 + |x|^3 & \text{for } 0 < |x| < 1 \\ 4 - 8|x| + 5|x|^2 - |x|^3 & \text{for } 1 < |x| < 2 \\ 0 & \text{otherwise} \end{cases}$$

The greyvalue at an arbitrary point [u, v] (black dot in figure) can be computed by

- 4 horizontal interpolations to obtain greyvalues at points [u, j-1] ... [u, j+2] (red dots), followed by
- 1 vertical interpolation (between red dots) to obtain greyvalue at [u, v].



cross section of interpolation kernel



Note:
For an image with constant geyvalues g_0 the interpolated greyvalues at all points between the grid lines are also g_0.

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