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MIN-Fakultät
Fachbereich Informatik
Arbeitsbereich SAV/BV (KOGS)

IP2: Image Processing in Remote Sensing
3. Electromagnetic Radiation I: Waves and Basic Principles

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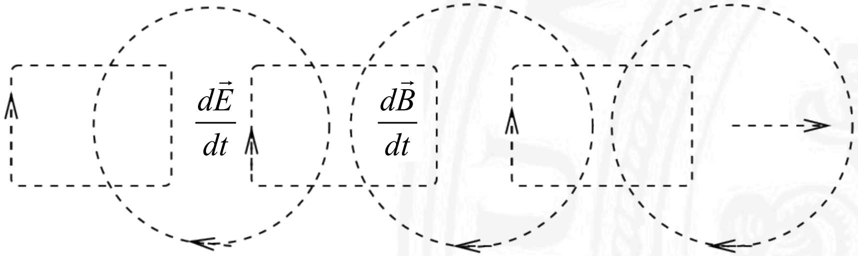
Agenda

- Electromagnetic Waves
- Harmonic Oscillation
- Interaction with Transmission Medium
- Wave packages
- The Doppler Effect
- Other Relativistic Effects

Electromagnetic Radiation

- EM Radiation is the most important transmission medium for Remote Sensing applications *Übertragungsmedium*
- Consists of electric and magnetic fields (Maxwell's Equations)
- Creation of spatial EM waves:
  - A temporal varying electric field creates a magnetic field
  - A temporal varying magnetic field creates an electric field

    → A traversal wave is created.



E: electric field strength *elektrische Feldstärke*

B: magnetic field intensity *magnetische Feldstärke*

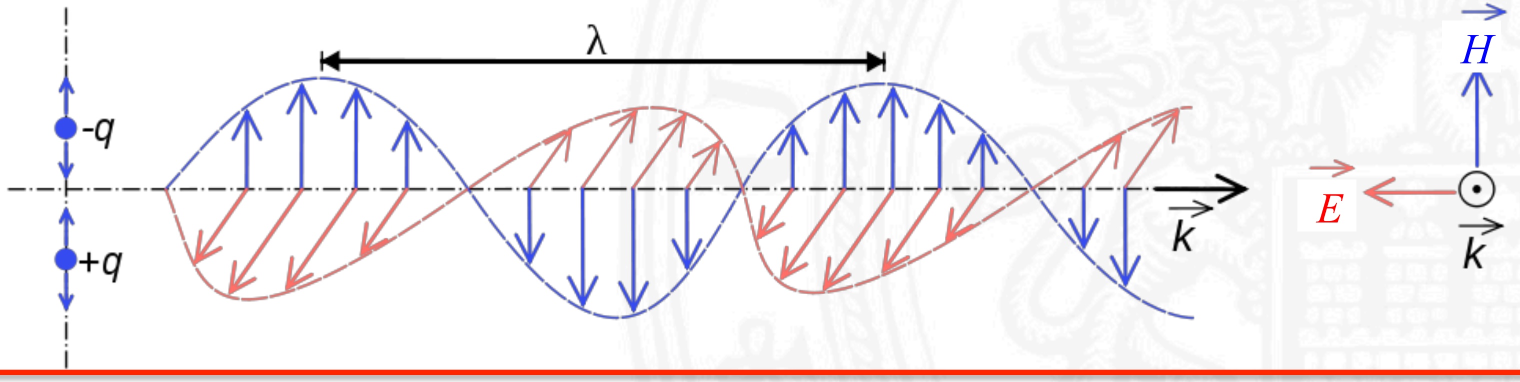
EM Spatial Waves

A planar EM wave consist of:

- An electric field, which oscillates perpendicular to the travelling direction k. It is described by the vector of electric field strength $\boldsymbol{E}$.
- A magnetic field, perpendicular to the electric field and oscillating perpendicular to the travelling direction k. It is described by the vector of magnetic field intensity $\boldsymbol{H}$.

Characterization of the EM wave:

- Wavelength: λ
- Frequency: v
- Velocity: $v = \lambda v$ (at vacuum: c)



Maxwell's Equations

Following Rees, 1990:

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{\mu_r \varepsilon_r}{c^2} \frac{\partial \vec{E}}{\partial t}$$

with:

Inductivity: $\vec{B} = \mu_0 \mu_r \vec{H}$

and material properties:

Dielectric constant: ε

Dielektrizitätskonstante: ε

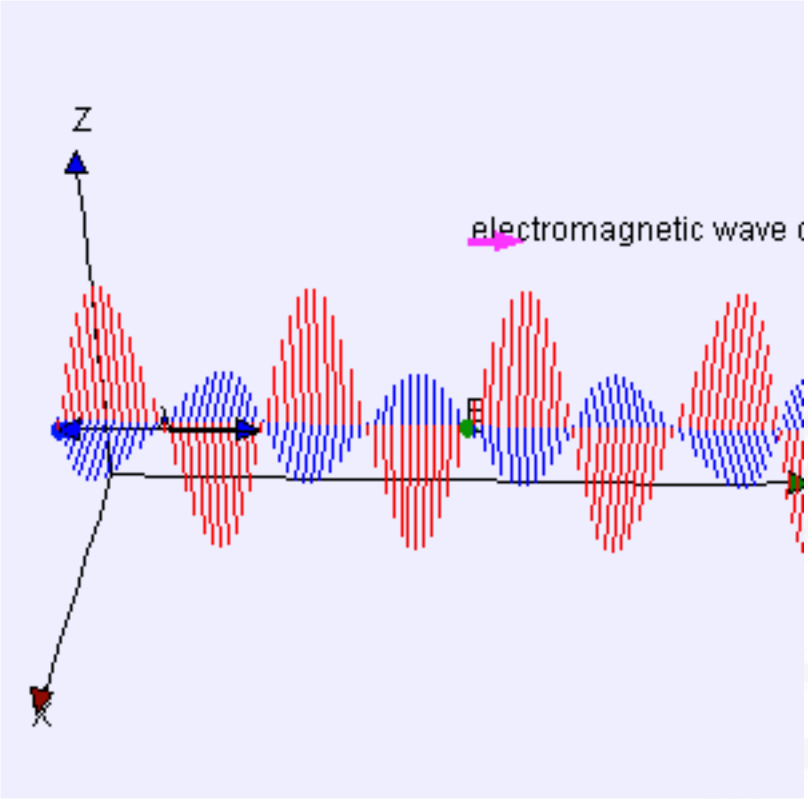
Permeability: μ

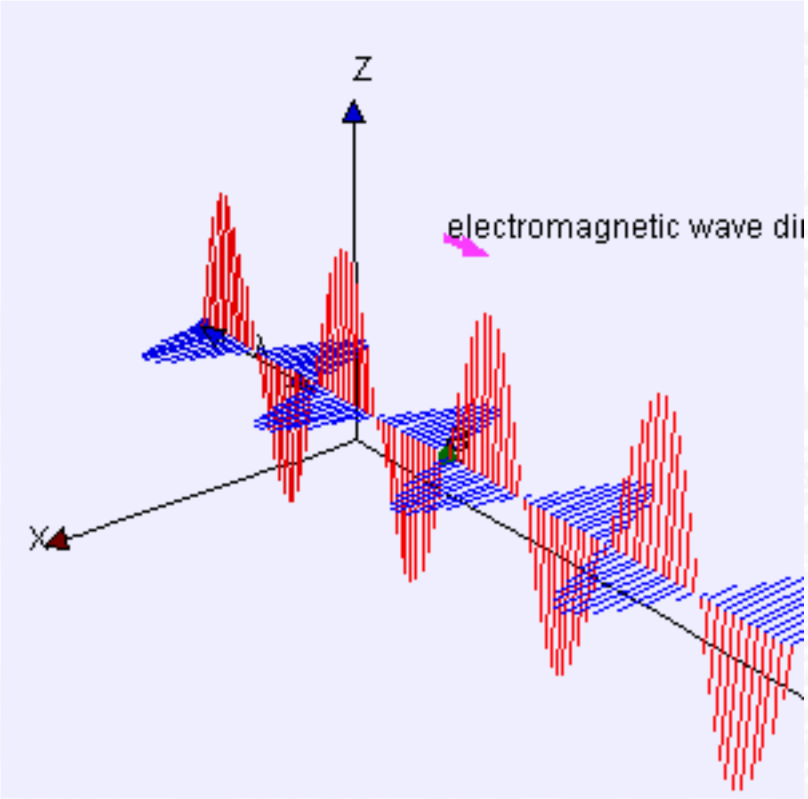
Permeabilität: μ

Notation for material property coefficients:

- Inside vacuum: ε_0 , μ_0
- Inside a medium (relative to vacuum): ε_r , μ_r or: ε , μ

Maxwell's Equations: Live-Demo





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General Properties of EM Waves

- Die planar spatial wave is a t*ransversal wave*.
- The electric field ***E*** is perpendicular to the magnetic field ***H*** (***B***) the travelling direction of the wave ***E***×***H*** (***E***×***B***) and is perpendicular to both fields
- The oscillation plane of the ***H*-vector** is called ***polarization plane*** of the wave.
- The oscillation plane of the ***E*-vector** is called **oscillation plane** of the wave..
- The vector ***E***×***B*** is called Pointing vector ***S***.
- The energy transport of the wave in performed in the direction of ***S***:

  |***S***| = ***E***·***B*** gives the energy, which is transported perpendicular to the travelling direction.
  (in time units per area units).

Planar EM Waves Equation

Given as a solution of Maxwell's Equations in homogeneous isotropic, non-magnetic media with the following wave equations:

$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \mu_r \varepsilon_r \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{B} - \mu_0 \varepsilon_0 \mu_r \varepsilon_r \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

with (partial) spatial derivatives:

$$\nabla^2 \vec{E}, \nabla^2 \vec{B}$$

and (partial) temporal derivatives:

$$\frac{\partial^2 \vec{E}}{\partial t^2}, \frac{\partial^2 \vec{B}}{\partial t^2}$$

Note: Temporal change of a wave at a location yields to a change of the wave in travelling direction.

Harmonic EM Waves (1)

For harmonic waves, the wave equation is given by:

$$\nabla^2 \vec{E} - \frac{\omega^2}{c_r^2} \vec{E} = 0$$

with:

$$c_r = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}}$$

$$v = c_r = \frac{c_0}{\sqrt{\mu_0 \varepsilon_0}}$$

$$c = c_0 = \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

Using Euler's Formula, we can express the wave function complex-valued:

$$\vec{E} = \vec{A} e^{i(kr - \omega t + \phi)}$$

with:

- $\vec{A}$ Amplitude
- ω Complex angular frequency
- t Time point
- k Wave vector at medium
- r Space point
- ϕ Phase

and:

$$k = \frac{2\pi \sqrt{e_r}}{\lambda}$$

$$\lambda = \frac{2\pi c_0}{\omega} \quad \text{Wavelength}$$

$$\nu = \frac{\omega}{2\pi} \quad \text{Frequency}$$

Euler's Formula

$$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$$

Harmonic EM Waves (2)

Another definition: The wave can be described by a sine in a three dimensional space: The electric field strength ***E*** is parallel to the x-axis, the vector ***B*** is parallel to the y-axis, the travelling direction of the wave z.

$$\vec{E}_y = \vec{E}_z = 0$$
$$\vec{E}_x = \vec{E}_0 \cos(\omega t - kz)$$
$$\vec{B}_x = \vec{B}_z = 0$$
$$\vec{B}_y = \vec{B}_0 \cos(\omega t - kz)$$

with:

$\omega = 2\pi\nu = \frac{ck}{n}$	Angular frequency
$k = \frac{2\pi}{\lambda}$	Wave number
$\nu = \frac{c}{\lambda n}$	Frequency
$v = \lambda\nu = \frac{c}{n}$	Phase velocity
$n = \sqrt{\varepsilon\mu}$	Refractivity *Brechungsindex*

- E_0, B_0 are amplitudes of field intensities
- kz space dependent variation of field intensity
- ωt space dependent variation of field intensity

The Transmission Medium

- General properties
- Impedance

  Wellenwiderstand, Impedanz
- Dielectric constant

  Dielektrizitätskonstante
  - *Absorbing media*
  - *Dispersing media*
  - *Plasma*

Transmission Medium: General properties

- c_r (or v) denotes the phase velocity of a wave inside the medium,
- c_0 (or c or C) $= 2.99792458 \times 10^8$ m/s the phase velocity of a wave inside vacuum.
- The permeability μ_r (usually) equals 1 (for non-magnetic media)
- The dielectric constant ε_r (usually) varies between 1 to 80. [Elachi, 1987]

  ε_r is a function of the frequency → dispersion!

Transmission Medium: Impedance

- The electric and the magnetic field intensity are oscillating with same phase, thus the ratio of both field intensities (the impedance) remains constant.
- The impedance Z is a characteristic constant of a medium.
- Z_0 denotes the „impedance at vacuum“.

$$Z = \frac{E}{H} = \frac{E}{B}\mu_r\mu_0 = \frac{E_0}{B_0}\mu_r\mu_0 = v \cdot \mu_r\mu_0$$

$$= \sqrt{\frac{\mu_r\mu_0}{\varepsilon_r\varepsilon_0}} = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

- Impedance is inverse proportional to the phase velocity of the wave: The higher the impedance, the slower the wave!

Transmission Medium: Dielectric constant

- **Remember:** The phase velocity v of a spatial wave depends on the dielectric constant ε of the medium:
  $$v = \frac{c}{n}, \quad n = \sqrt{\varepsilon\mu}$$
- Assuming $\mu = 1$ it follows that $n = \sqrt{\varepsilon}$
- **Attention!** Only valid for lossless transmission media – the wave needs to pass the medium without loss of energy.

Dielectric Constant in Absorbing Media

- In absorbing media, the dielectric constant ε is complex valued:

  $$\varepsilon = \varepsilon' - i \cdot \varepsilon''$$

- And so is the refractivity:

  $$n = n' - i \cdot \kappa$$

- Alternate notation (with $\tan(\delta)$ - loss tangent):

  $$n = \varepsilon'\left(1 - i \cdot \tan(\delta)\right)$$

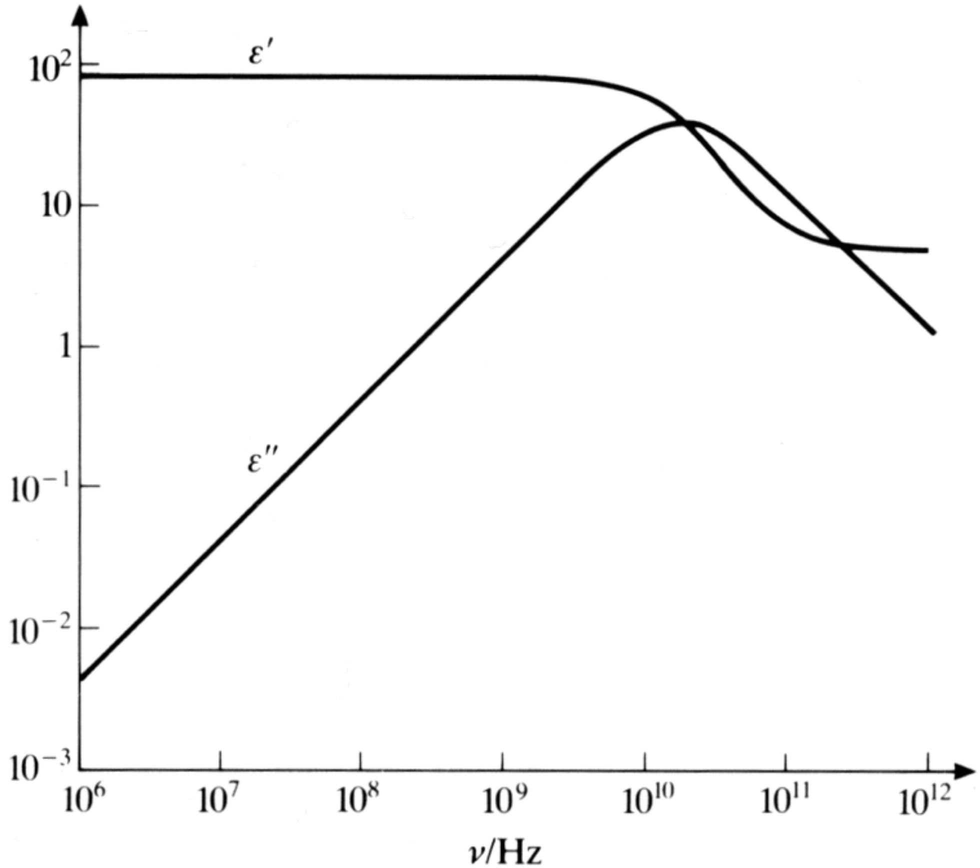
  Verlusttangens

Dielectric Constant in Dispersive Media

- In non-polarizing media, ε' and ε'' are constant.
- If the medium is polarizing, ε' and ε'' are (as well as the refractivity n) depending on the frequency of the wave.
  → Dispersion, e.g. water, glass
- The frequency dependency of dispersive media is expressed by the Debye Equation (resonance effects, see Rees 1990)
- In electrically conducting medias, the imaginary component of the refraction index depends on the conductivity of the material.

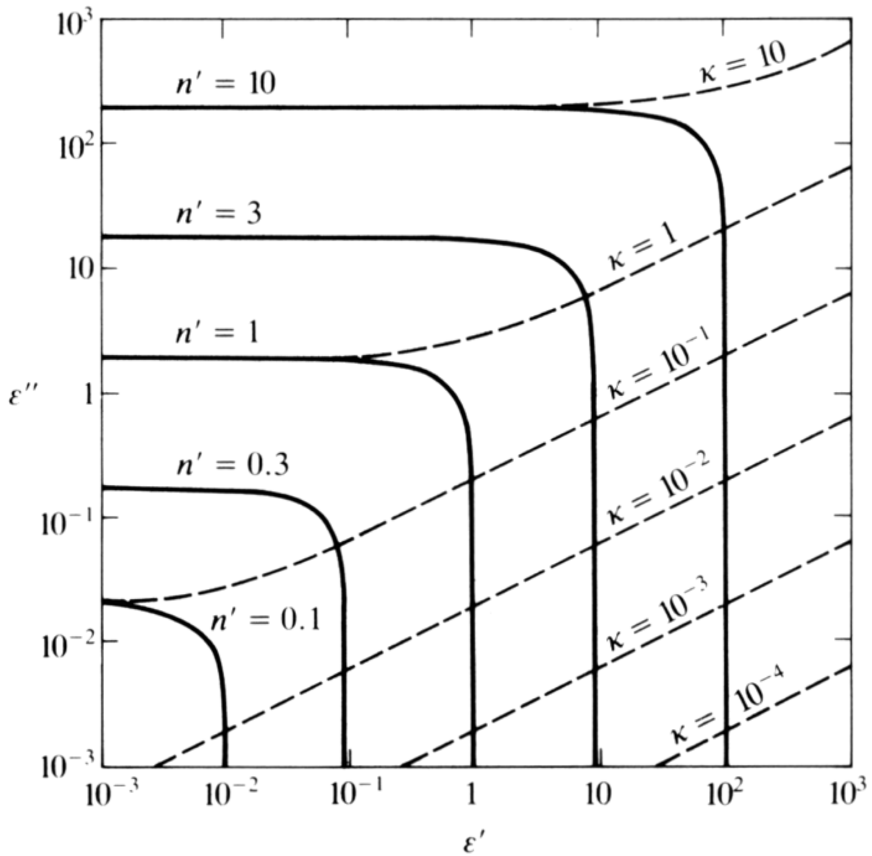
Example: Refraction Index of Water

Fig. 2.2. The real and imaginary parts of the dielectric constant of pure water at 20 °C.



Dielectric constant vs. refraction index

Fig. 2.3. The relationship between dielectric constant ($\varepsilon' - i\varepsilon''$) and refractive index ($n' - i\kappa$).



Dielectric Constant in Plasma

- In plasma (e.g. inside the Ionosphere) all atoms are ionized.
- Let:

  N be the count of ions per volume unit

  m be the mass of the particles:

$$\varepsilon = n^2 = 1 - N \frac{e^2}{\varepsilon_0 m \omega^2}$$

Observations:

- n is real valued for high frequencies and
  imaginary valued fro low frequencies
- ε may become smaller than 1 .
  → The phase velocity becomes larger than c (speed of light)!

Waves Packages

Wellengemische, Wellenpakete

From single waves → wave packages:

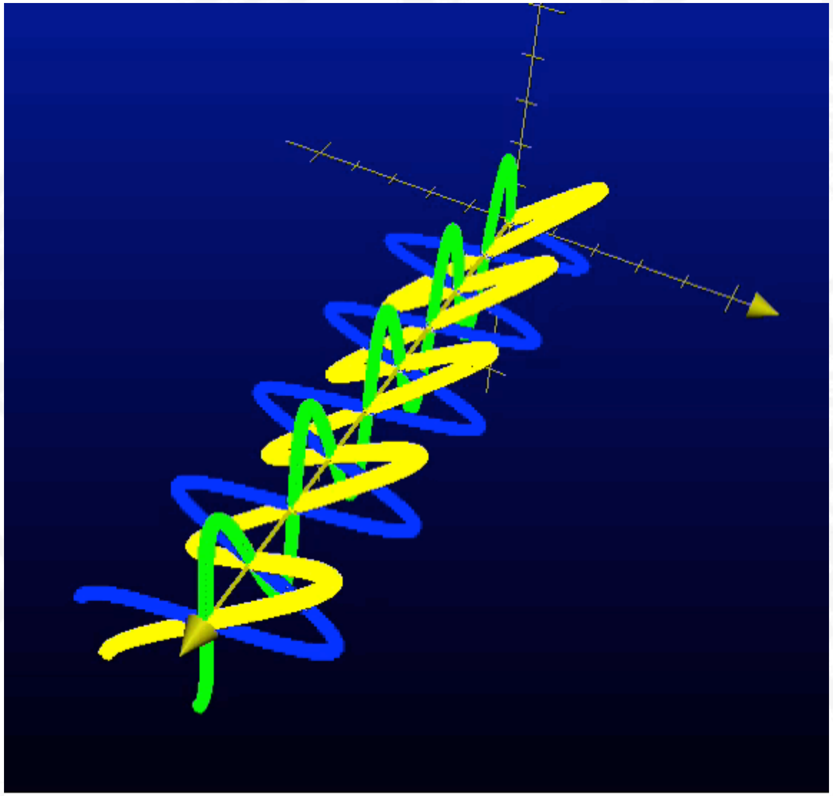
- Polarization
  - Linear
  - Circular
  - Elliptic
- Coherence
- Wave packages' velocities
  - Phase velocity
  - Group velocity

Linear Polarization

An EM wave is linear polarized, if the electric field E oscillated within a fixed plane.

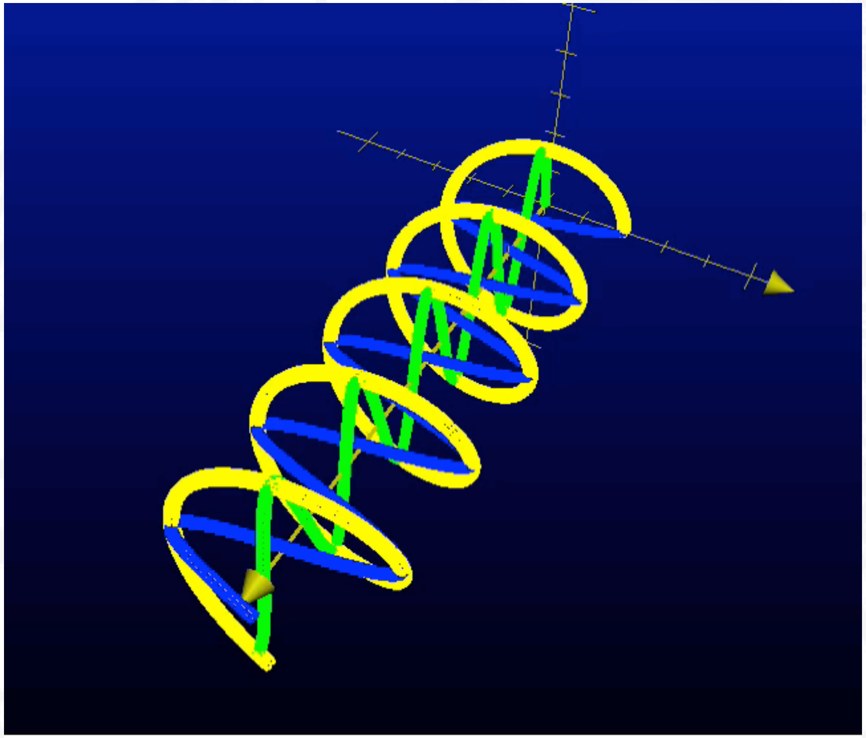
- **Horizontal Polarization:**
  The vector ***E*** is perpendicular to the plane of incidence of the radiation.

- **Vertical Polarization:**
  The vector ***E*** lies within the plane of incidence of the radiation.



Circular Polarization

- The superposition of two polarized waves of **equal** phase and wavelength yields to a new **linear polarized** wave. The polarization plane is given by the addition of both electric fields.
- The superposition of two linear polarized waves of **different** polarization and phase yields to a **circular polarized wave**, with a rotating direction of the electric field



Further Polarization Properties

- The relative phase of both components determines if the resulting wave is:
  - Circular (shape of the helix is a circle)
  - Elliptic (shape of the helix is an ellipse) polarized
- Depending on the rotational direction is the wave called:
  - Left-hand (LHC) polarization (counterclockwise)
  - Right-hand (RHC) polarization (clockwise)
- Radiation is named non-polarized, if the polarization randomly varies
- Some sensors are only sensitive to certain polarization schemes, e.g. H- and V-Glasses for 3D movies at cinema

Coherence

The coherence of a wave package is the time, which passes between the amplitudes of strong correlations. More precisely:

- For two waves with frequencies ν and $\nu + \Delta\nu$ the coherence time Δt is defined as the amount of time, after which the phases are shifted one cycle at each other:

$$\nu \Delta t + 1 = (\nu + \Delta \nu)\Delta t$$

$$\Rightarrow \Delta \nu \Delta t = 1$$

$$\Rightarrow \Delta t = \frac{1}{\Delta \nu}$$

- The coherence length is defined as:

$$\Delta l = c\,\Delta t = \frac{c}{\Delta \nu}$$

Phase and Group Velocity

- The phase velocity v_p of a wave is the velocity at which the phase of the wave propagates in space:

$$v_p = \frac{\omega}{k}$$

- Sensors (often) do not measure the phase velocity but the group velocity v_g, which is the velocity of pulses, which are modulated on carrier waves:

$$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k}$$

  - The modulations results in additional frequencies. In dispersive media $v_g < v_p$ holds.
  - $\mathrm{v_g}$ is the velocity, which is used to transport energy. Thus $v_g \leq c$ holds, too.

Phase and Group Velocity (2)

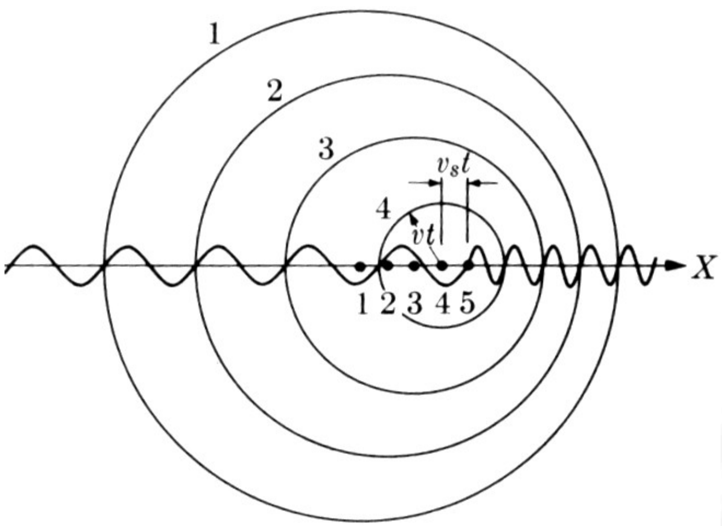
no dispersion

dispersion

Doppler Effect

- The result of the observation of a wave's frequency depends on the velocity and direction of the observer, the sender and the transmission medium:
  - Moving sender
  - Moving observer
  - Moving reflecting, scattering body
- For large velocities (close to c), the Doppler Effect needs to be computed with the equations of the special relativity.
- For spatial EM waves, the Doppler effect depends only on the relative velocity between observer and sender
- For acoustic or surface waves other equations may be used for computation, depending on the motion of observer or sender or both.

Moving Sender



(a)

- The apparent wavelength is shortening in motion direction.
- The apparent wavelength is increasing opposite to the motion direction.
- Reason: velocity of sender adds to the velocity of the wave.
- For an observer from the side, only the velocity component of the sender in direction to the observer is “visible”.

Doppler Effect for EM Radiation

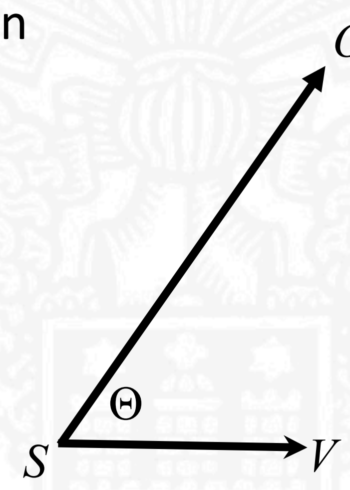
High velocities require relativistic formulas:

$$\frac{\nu'}{\nu} = \frac{\sqrt{1-\frac{V^2}{c^2}\cos^2(\Theta)}}{1-\frac{V}{c}\cos(\Theta)}$$

with:
- ν emitted frequency
- ν' observed frequency
- V relative velocity of sender S and observer O
- Θ Angle between motion and observation direction

(Very) small velocities: non-relativistic formula:

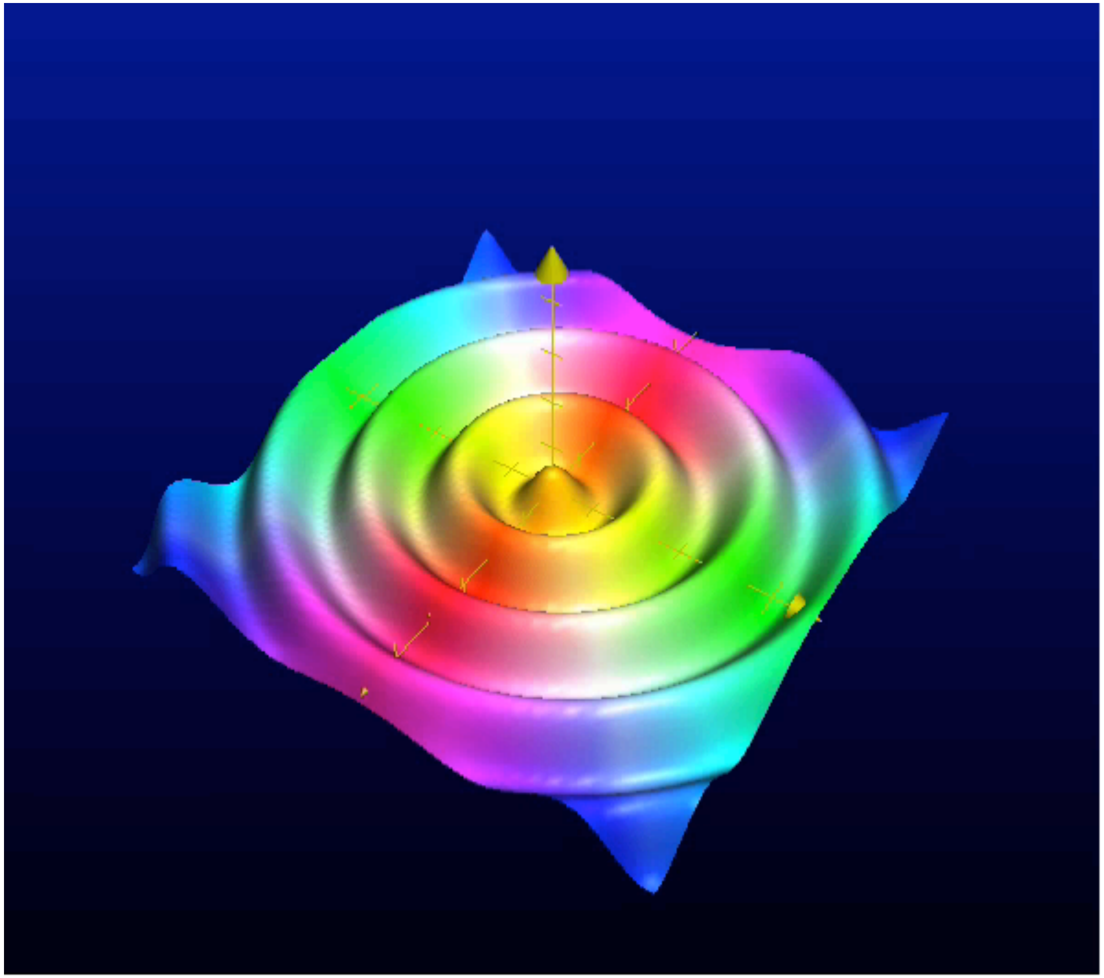
$$\frac{\nu'}{\nu} = 1 + \frac{V}{c}\cos(\Theta)$$



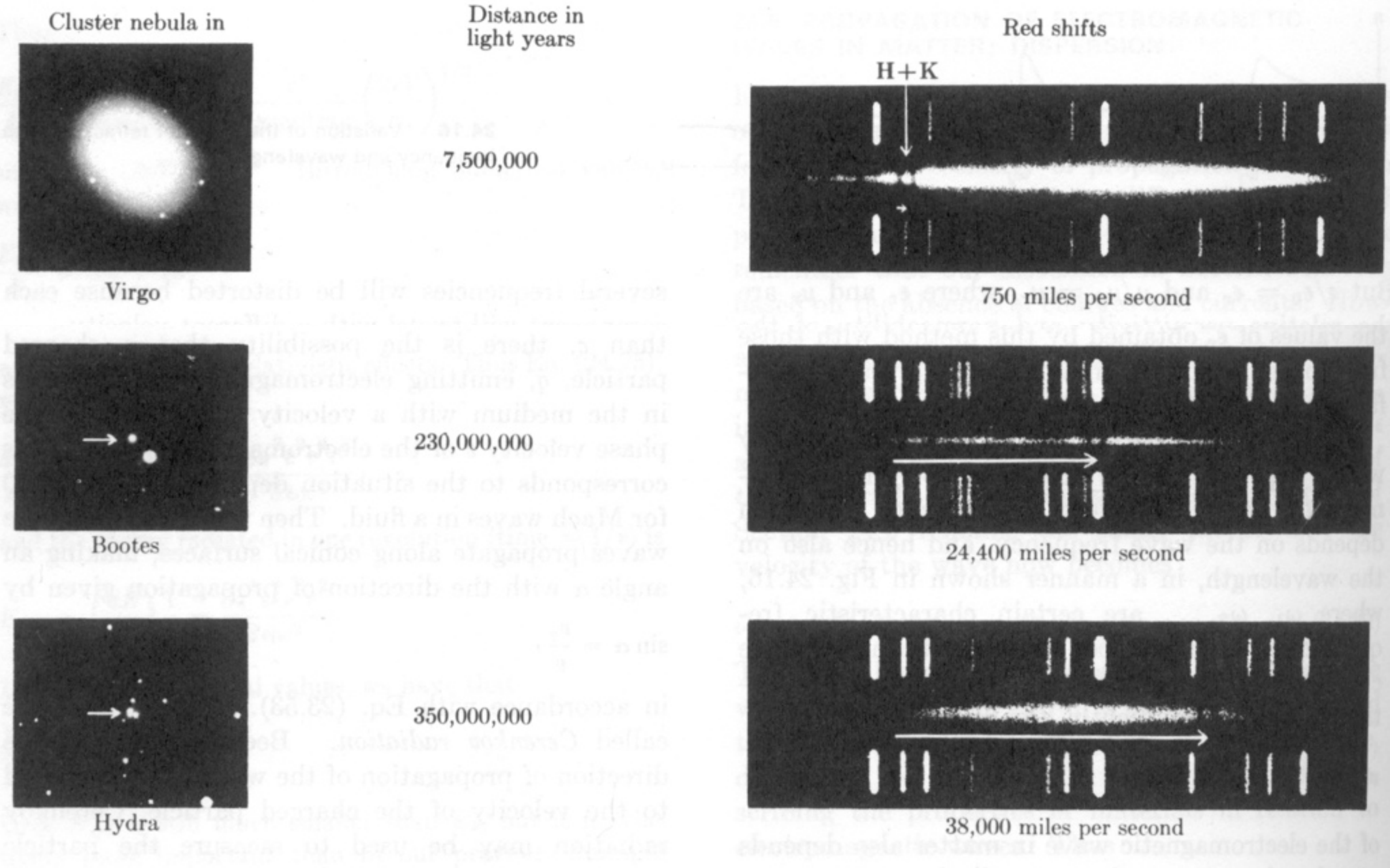
Example:

Satellite V = 7km/s, Θ=5°, ν=5GHz → ν-ν'=116 kHz

Doppler Effect: 0.1c - 0.9c



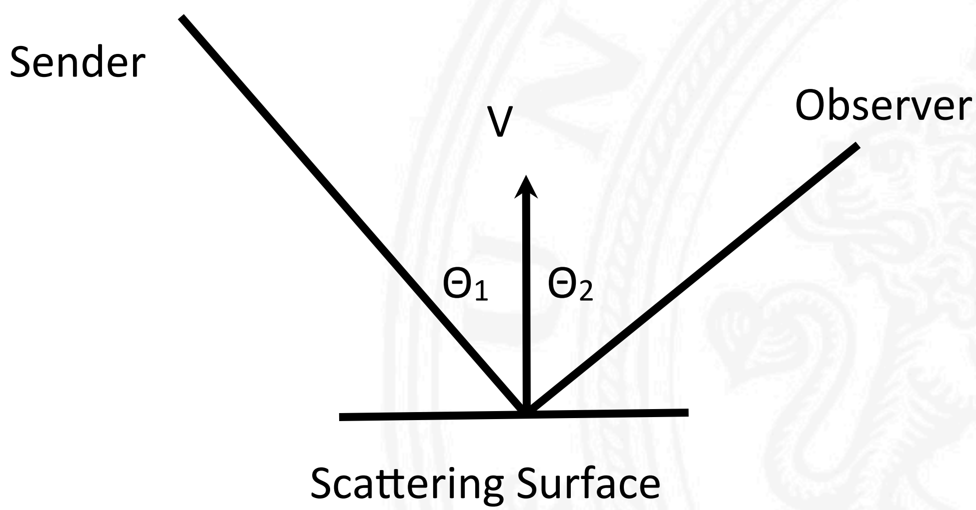
Relativistic Doppler Effect: The Expanding Universe



24.17 Doppler effect in extragalactic nebulas. The red shift of the spectral H and K calcium lines (indicated by the arrow) increases with the distance of the nebula, suggesting greater recessional velocities. [Photograph courtesy of Mt. Wilson and Palomar Observatories.]

Moving Reflector and Scattering (1)

- The Doppler Effect also appears, if neither the signal source nor the sensor is moving!
  Precondition: the wave is scattered by a third object or is directly reflected from a moving object!
- Typical situation in RADAR-based Remote Sensing



Moving Reflector and Scattering (2)

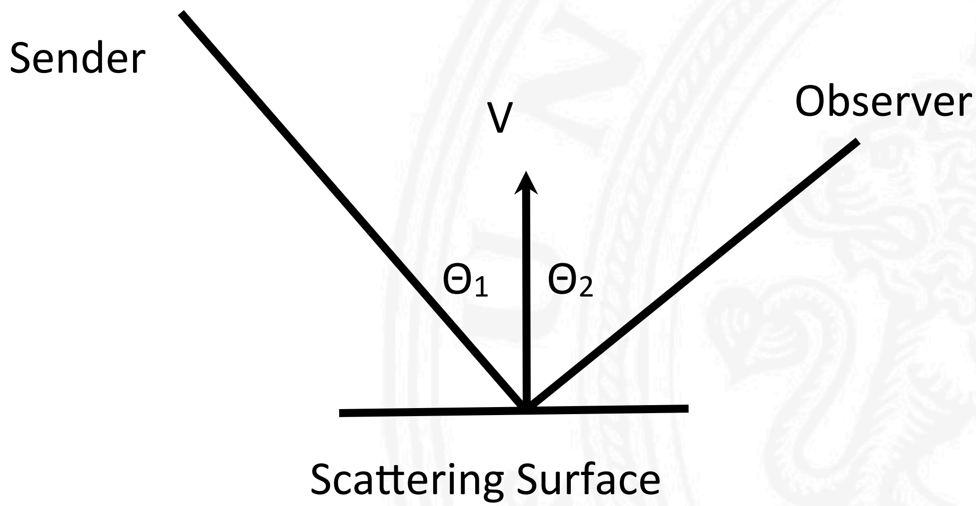
Computation of the increasing rate of the frequency νd

- Observer and sender at different locations:

$$\nu d = \nu\left(\cos(\Theta_1) + \cos(\Theta_2)\right)\frac{v}{c}$$

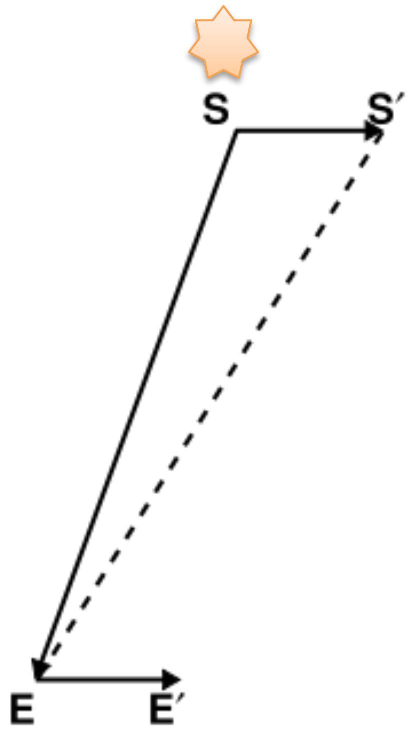
- Observer and sender at the same location:

$$\Theta_1 = \Theta_2 = \Theta \Rightarrow \nu d = 2\nu\cos(\Theta_1)\frac{v}{c}$$



Aberration of Light

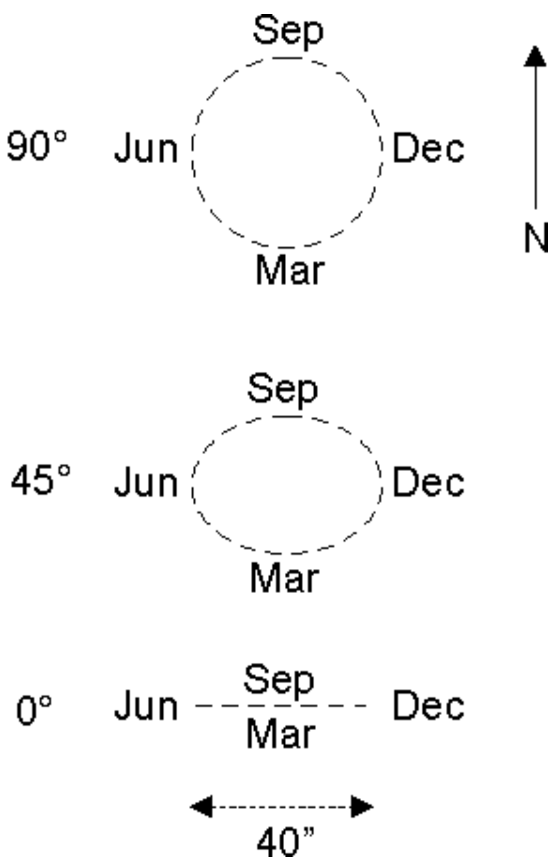
Abberation des Lichts



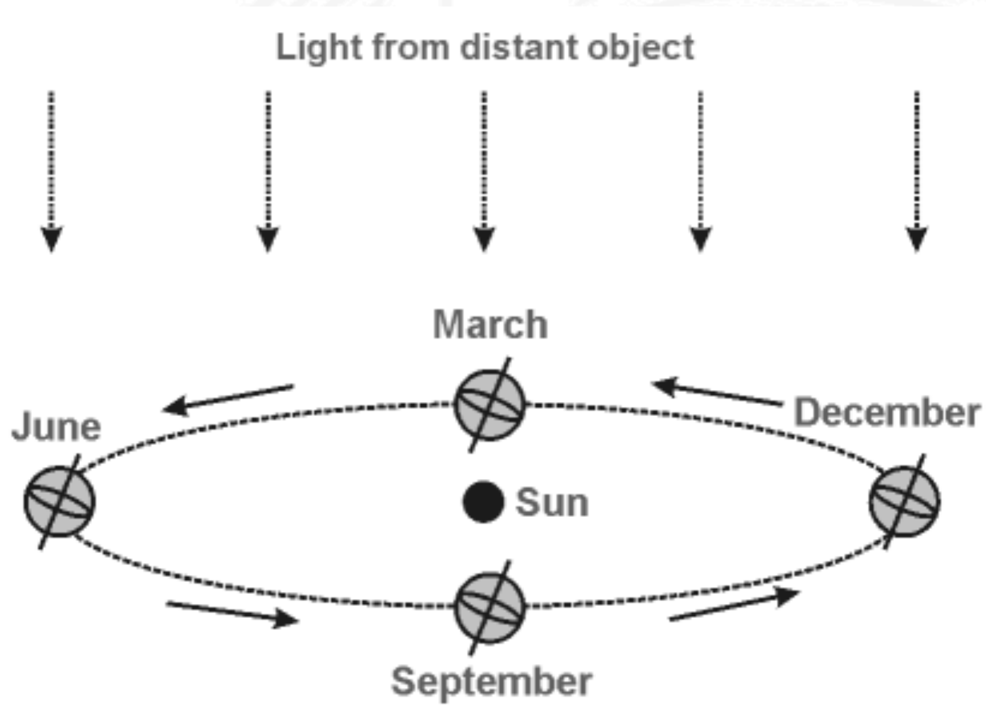
If observer and light source are moving relative to each other:

- **not only the apparent frequency is changing** but
- the apparent position of the light source is changing, too!

The Seasonal Aberration



Mean change ~ 20’’ (arc seconds) difference from the “real” position.



Time of Flight

- For position measurements at large distances, the time of flight (of light) needs to be taken into account.
- We measure the position at the time of their emission, not at the arrival of the light.
- This can yield to large effects w.r.t. planetary space probes:
- Exemplary Runtimes:
  - Earth → Moon: ca. 1 second
  - Earth → Sun: ca. 8.2 minutes