Universität Hamburg
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MIN-Fakultät
Fachbereich Informatik
Arbeitsbereich SAV/BV (KOGS)

Image Processing 1 (IP1)
Bildverarbeitung 1

Lecture 6 – Image Properties and Filters

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Global Image Properties

Global image properties refer to an image as a whole rather than components. Computation of global image properties is often required for image enhancement, preceding image analysis.

We treat

- empirical mean and variance
- histograms
- projections
- cross-sections
- frequency spectrum

Empirical Mean and Variance

Empirical mean = average of all pixels of an image

$$\bar{g} = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} g_{mn} \quad \text{with image size: } M \times N$$

Simplified notation: $\bar{g} = \frac{1}{K}\sum_{k=0}^{K-1} g_k$

Incremental computation: $\bar{g}_0 = 0 \qquad \bar{g}_k = \frac{\bar{g}_{k-1}(k-1) + g_k}{k} \quad \text{with } k = 2 \ldots K$

Empirical variance = average of squared deviation of all pixels from mean

$$\sigma^2 = \frac{1}{K}\sum_{k=1}^{K}(g_k - \bar{g})^2 = \frac{1}{K}\sum_{k=1}^{K} g_k^2 - \bar{g}^2$$

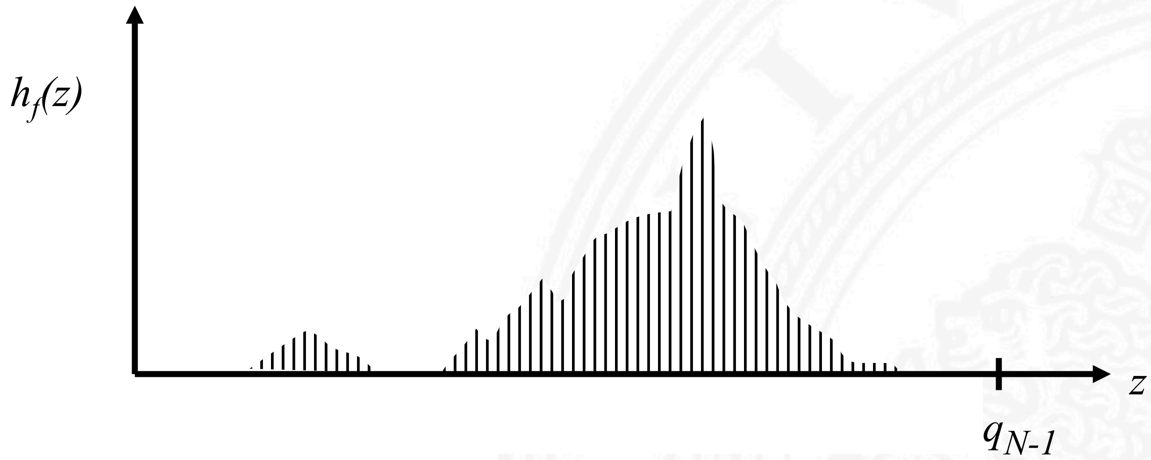
Incremental computation:

$$\sigma_0^2 = 0 \qquad \sigma_k^2 = \frac{\left(\sigma_{k-1}^2 + \bar{g}_{k-1}^2\right)(k-1) + g_k^2}{k} - \left(\frac{\bar{g}_{k-1}(k-1) + g_k}{k}\right)^2 \quad \text{with } k = 2 \ldots K$$

Greyvalue Histograms

A greyvalue histogram $h_f(z)$ of an image f provides the frequency of greyvalues z in the image.

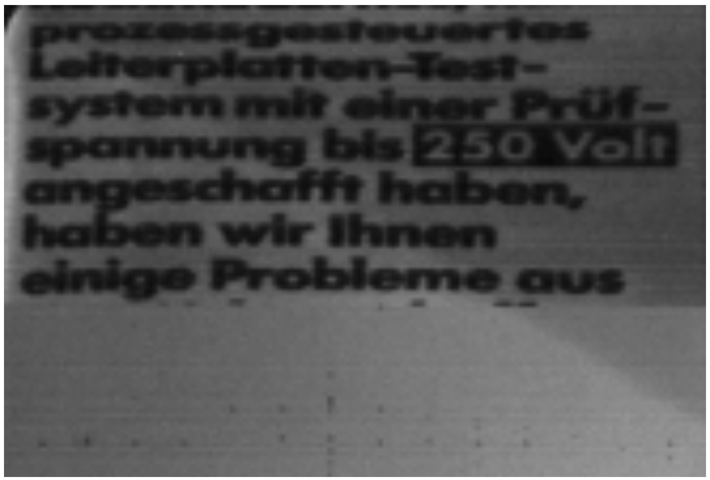
The histogram of an image with N quantization levels is represented by a 1D array mit N elements.



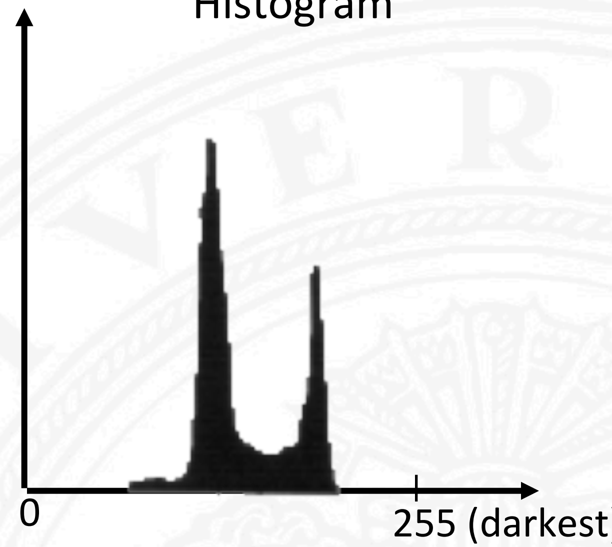
A greyvalue histogram describes discrete values, a greyvalue distribution describes continuous values.

Example of Greyvalue Histogram

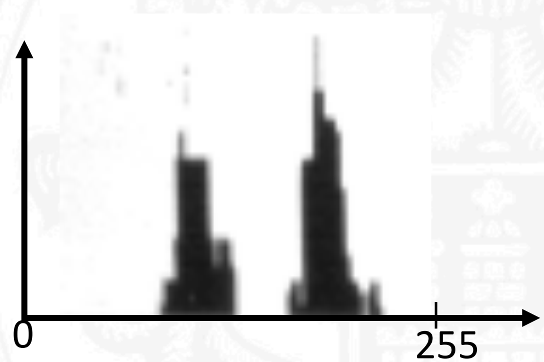
Image



Histogram



A histogram can be "sharpened" by discounting pixels at edges (more about edges later):



Histogram Modification (1)

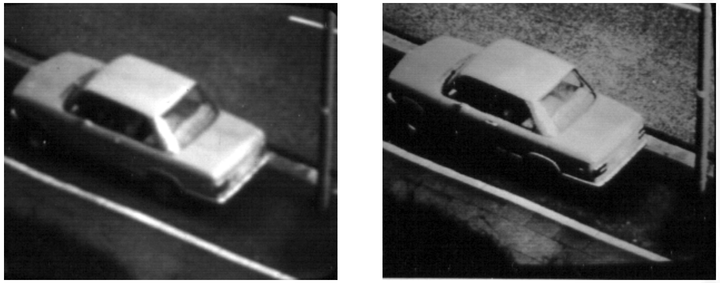
Greyvalues may be remapped into new greyvalues to

- facilitate image analysis
- improve subjective image quality

Example: Histogram equalization

1. Cut histogram into N stripes of equal area (N = new number of greyvalues)

2. Assign new greyvalues to consecutive stripes



Examples show improved resolution of image parts with most frequent greyvalues (road surface)

Histogram Modification (2)

Two algorithmic solutions:

N pixels per image, greyvalues $i=0...255$, histogram $h(i)$

1. "Cutting up a histogram into stripes of equal area"

stripe area $S = N/256$

Cutting up an arbitrary histogram into equal stripes may require assignment of different new greyvalues to pixels of the same old greyvalue.

2. Gonzales & Woods "Digital Image Processing" (2nd edition)

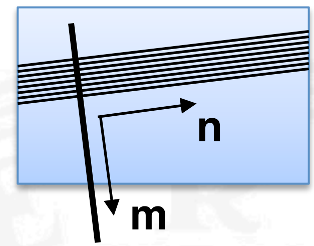
old greyvalue i, new greyvalue $k = T(i)$

Transformation function $$T(i) = \text{round}\left(255 \sum_{j=0}^{i} \frac{h(j)}{N}\right)$$

Histogram is only coarsely equalized.

Projections

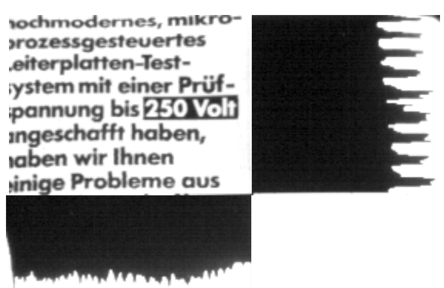
A projection of greyvalues in an image is the sum of all greyvalues orthogonal to a base line:

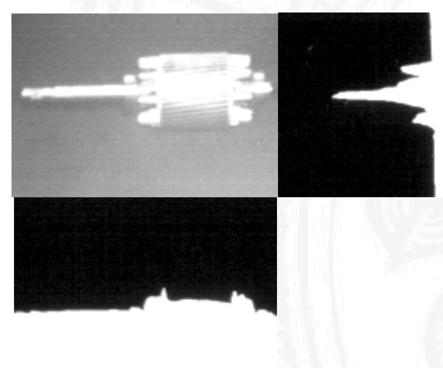


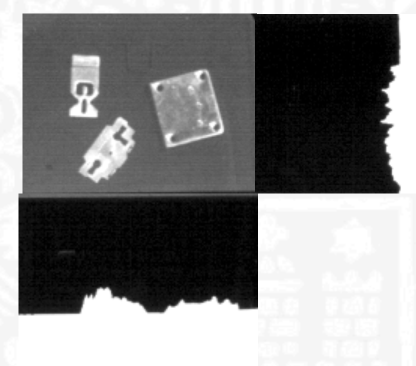
Often used:

$$p_m = \sum_n g_{mn}$$

- "row profile" = row vector of all (normalized) column sums
- "column profile" = column vector of all (normalized) row sums



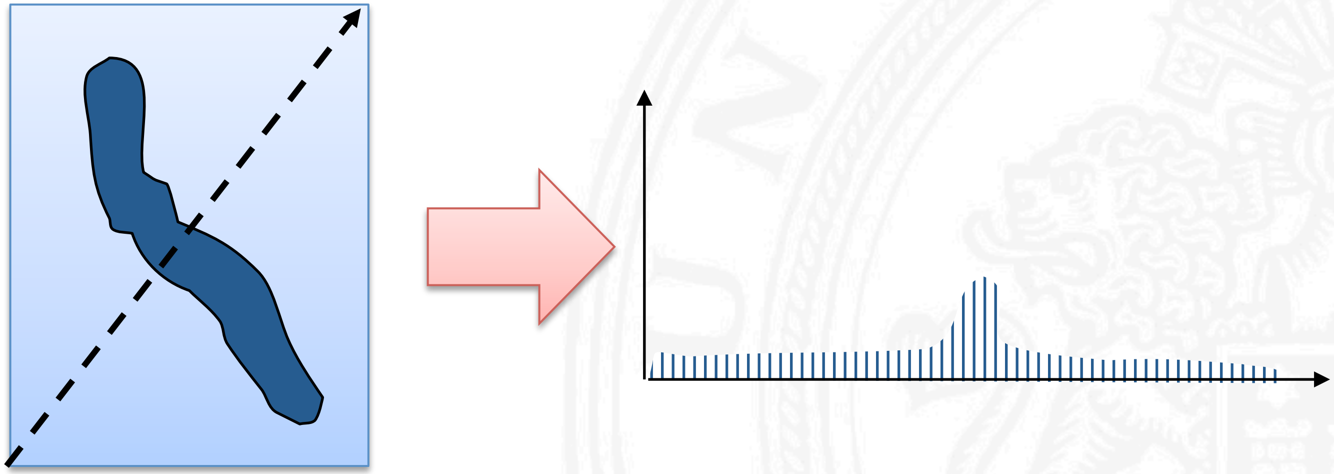




Cross-sections

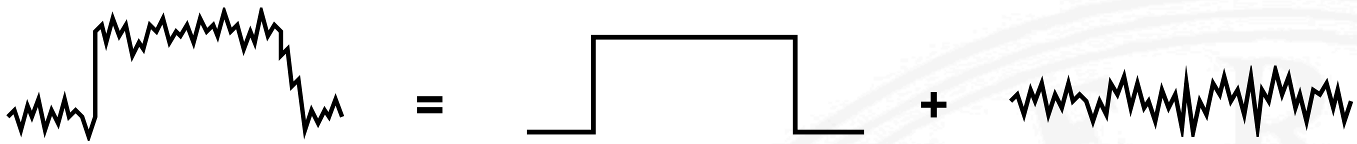
A cross-section of a greyvalue image is a vector of all pixels along a straight line through the image.

- fast test for localizing objects
- commonly taken along a row or column or diagonal



Noise

Deviations from an ideal image can often be modelled as additive noise:



Typical properties:

- mean 0, variance $\sigma^2 > 0$
- spatially uncorrelated: $E[\, r_{ij}\, r_{mn}] = 0$ for $ij \neq mn$
- temporally uncorrelated: $E[\, r_{ij,t1}\, r_{ij,t2}] = 0$ for $t1 \neq t2$
- Gaussian probability density: $p(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{r^2}{2\sigma^2}}$

$E[x]$ is "expected value" of x

Noise arises from analog signal generation (e.g. amplification) and transmission.

There are several other noise models other than additive noise.

Noise Removal by Averaging

Principle: $\hat{r}_K = \frac{1}{K}\sum_{k=1}^{K} r_k \Rightarrow 0$ sample mean approaches density mean

There are basically 2 ways to "average out" noise:

- temporal averaging if several samples $g_{ij,t}$ of the same pixel but at different times $t = 1 \ldots T$ are available
- spatial averaging if $g_{mn} \approx g_{ij}$ for all pixels g_{mn} in a region around g_{ij}

How effective is averaging of K greyvalues?

$\hat{r}_K = \frac{1}{K}\sum_{k=1}^{K} r_k$ is random variable with mean and variance depending on K

$$E\left[\hat{r}_K\right] = \frac{1}{K}\sum_{k=1}^{K} E\left[r_k\right] = 0 \quad \text{mean}$$

$$E\left[(\hat{r}_K - E\left[\hat{r}_K\right])^2\right] = E\left[\hat{r}_K^2\right] = E\left[\frac{1}{K^2}(\sum_{k=1}^{K} r_k)^2\right] = \frac{1}{K^2}\sum_{k=1}^{K} E\left[r_k^2\right] = \frac{\sigma^2}{K} \quad \text{variance}$$

Example: In order cut the standard deviation in half, 4 values have to averaged

Example of Averaging

Intensity averaging with 5 x 5 mask

$$\frac{1}{25}\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}$$

Simple Smoothing Operations

Example of 3-by-3 region D

	ij	

1. Averaging

$$\hat{g}_{ij} = \frac{1}{|D|} \sum_{g_{mn} \in D} g_{mn} \quad \text{with } D \text{ the set of all greyvalues around } g_{ij}$$

2. Removal of outliers

$$\hat{g}_{ij} = \begin{cases} \frac{1}{|D|} \sum_{g_{mn} \in D} g_{mn} & \text{if } \left| g_{ij} - \frac{1}{|D|} \sum_{g_{mn} \in D} g_{mn} \right| \geq S \quad \text{with threshold } S \\ g_{ij} & \text{else} \end{cases}$$

3. Weighted average

$$\hat{g}_{ij} = \frac{1}{\sum w_k} \sum_{g_k \in D} w_k g_k \quad \text{with } w_k = \text{weights in } D$$

Example of weights in 3-by-3 region

1	2	1
2	3	2
1	2	1

Note that these operations are heuristics and not well founded!

Bimodal Averaging

To avoid averaging across edges, assume bimodal greyvalue distribution and select average value of modality with largest population.

Determine:

$$\bar{g}_D = \frac{1}{|D|} \sum_{g_{mn} \in D} g_{mn}$$

$$A = \{g_k \text{ with } g_k \geq \bar{g}_D\} \qquad B = \{g_k \text{ with } g_k < \bar{g}_D\}$$

$$g'_D = \begin{cases} \frac{1}{|A|} \sum_{g_k \in A} g_k & \text{if } |A| \geq |B| \\ \frac{1}{|B|} \sum_{g_k \in B} g_k & \text{else} \end{cases}$$

Example:

B

11	14	15
13	12	25
15	19	26

A

$\bar{g}_D = 16.7$ → *A, B* → $g'_D = 13$

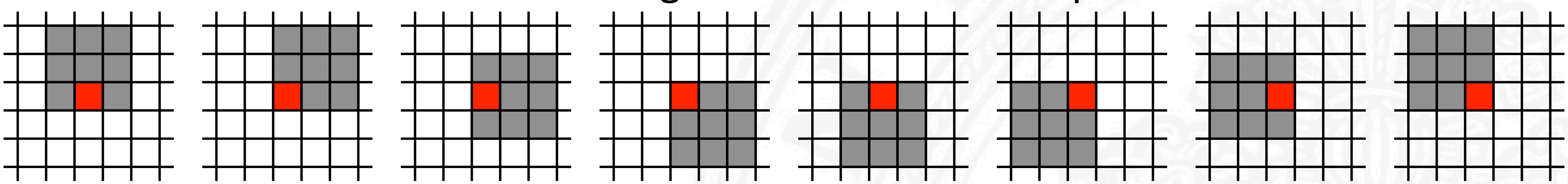
Averaging with Rotating Mask

Replace center pixel by average over pixels from the most homogeneous subset taken from the neighbourhood of center pixel.

Measure for (lack of) homogeneity is dispersion σ^2 (= empirical variance) of the greyvalues of a region D:

$$\bar{g}_{ij} = \frac{1}{|D|} \sum_{g_{mn} \in D} g_{mn} \qquad \sigma_{ij}^2 = \frac{1}{|D|} \sum_{g_{mn} \in D} (g_{mn} - \bar{g}_{ij})^2$$

Possible rotated masks in 5 x 5 neighbourhood of center pixel:



<u>Algorithm</u>:

1. Consider each pixel g_{ij}
2. Calculate dispersion in mask for all rotated positions of mask
3. Choose mask with minimum dispersion
4. Assign average greyvalue of chosen mask to g_{ij}

Median Filter

Median of a distribution $P(x)$: x_m such that $P(x < x_m) = 1/2$

$$\hat{g}_{ij} = \max(a) \text{ with } g_k \in D \text{ and } \left|\{g_k < a\}\right| < \frac{|D|}{2}$$

<u>Median Filter</u>:

1. Sort pixels in D according to greyvalue
2. Choose greyvalue in middle position

<u>Example:</u>

11	14	15
13	12	25
15	19	26

11
12
13
14
15
15
19
25
26

greyvalue of center pixel of region is set to 15

Median Filter reduces influence of outliers in either direction!

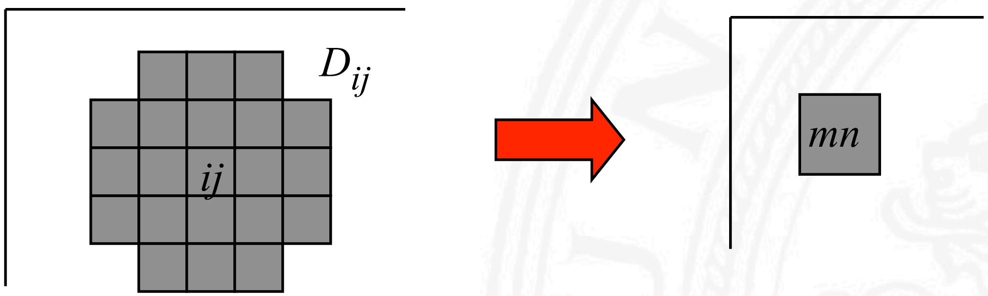
Local Neighbourhood Operations

Many useful image transformations may be defined as an instance of a local neighbourhood operation:

Generate a new image with pixels $\hat{g}_{mn}$ by applying operator f to all pixels g_{ij} of an image

$$\hat{g}_{mn} = f(g_1, g_2, ..., g_K) \qquad g_1, g_2, ..., g_K \in D_{ij}$$

example of neighbour-hood



Pixel indices i, j may be incremented by steps larger than 1 to obtain reduced new image.

Example of Sharpening

intensity sharpening
with 3 x 3 mask

-1	-1	-1
-1	9	-1
-1	-1	-1

"unsharp masking" =
subtraction of blurred image

$$\hat{g}_{ij} = g_{ij} - \frac{1}{|D|} \sum_{g_{mn} \in D} g_{mn}$$