Universität Hamburg
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MIN-Fakultät
Fachbereich Informatik
Arbeitsbereich SAV/BV (KOGS)

Image Processing 1 (IP1)
Bildverarbeitung 1

Lecture 20: Shape from Shading

Winter Semester 2014/15

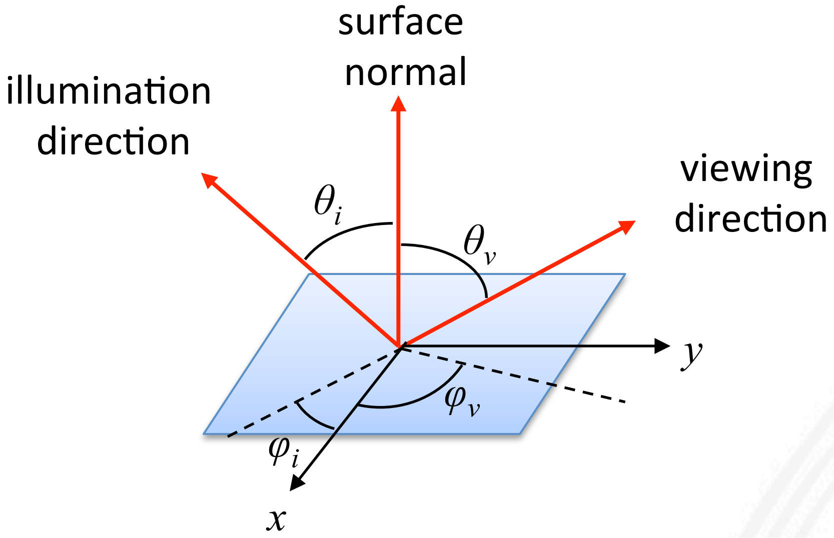
Dr. Benjamin Seppke
Prof. Siegfried Stiehl

Obtaining 3D Shape from Shading Information

From "Shape from Shading", B.K.P. Horn and M.J. Brooks (eds.), MIT Press 1989

By assuming certain conditions, a 3D surface model may be reconstructed from the greyvalue variations of a monocular image.

Reminder: Photometric Surface Properties



θ_i, θ_v polar (zenith) angles

φ_i, φ_v azimuth angles

In general, the ability of a surface to reflect light is given by the Bi-directional Reflectance Distribution Function (BRDF) r:

$$r(\theta_i, \phi_i; \theta_v, \phi_v) = \frac{\partial L(\theta_v, \phi_v)}{\partial E(\theta_i, \phi_i)}$$

radiance of surface patch towards viewer

irradiance of light source received by the surface patch

For many materials the reflectance properties are rotation invariant, in this case the BRDF depends on θ_i, θ_v, φ, where $\varphi = \varphi_i - \varphi_v$.

Units in Radiometry and Photometry

Radiometry	Photometry
Radiometry: branch of Physics	**Photometry**: closely related to radiometry, but studies human sensation
radiant flux Φ $[W]$ "radiant power"	**luminous flux** Φ_{ph} $[lm$ (= lumen)]

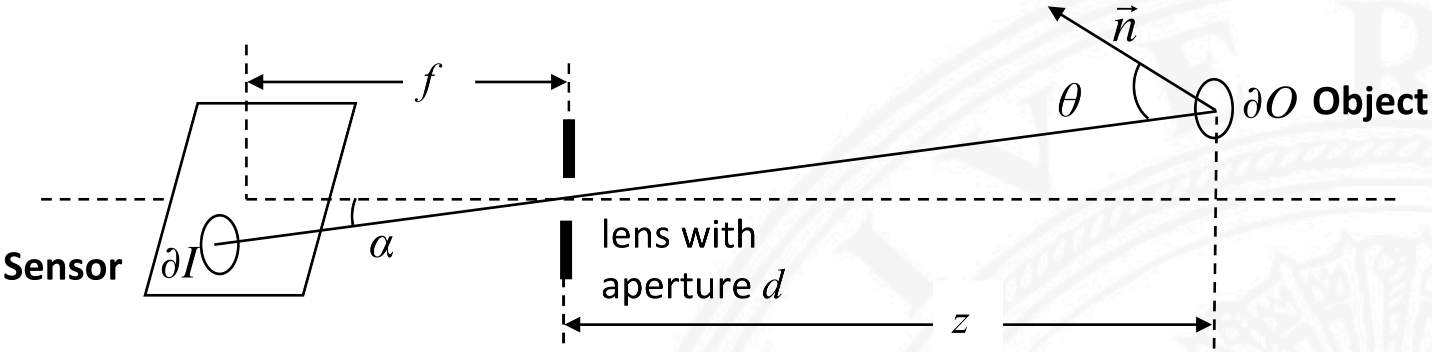
spatial angle Ω = area on unit sphere of cone with apex in center of sphere

- spatial angle of half sphere = 2π
- R distance between A and origin, $R^2 \gg A$, θ between area normal and vector from origin to A: $\Omega = \dfrac{A\cos\theta}{R^2}$

Radiometry	Photometry
irradiance E $[W\,m^{-2}]$ = power of light on unit area of object surface	**illumination** $[lm\,m^{-2}]$ = photometric equivalent to irradiance
radiance L $[W\,m^{-2}\,sr^{-1}]$ = power of light emit. from area into some spatial angle	**brightness** L_{ph} $[L\,m^{-2}\,sr^{-1}]$ = photometric unit equivalent to irradiance

Irradiance of Imaging Device I

irradiance = light energy falling on unit patch of imaging sensor,
sensor signal is proportional to irradiance



Sensor patch receives irradiance E,

has spatial angle $\frac{\partial I \cos\alpha}{(f/\cos\alpha)^2}$

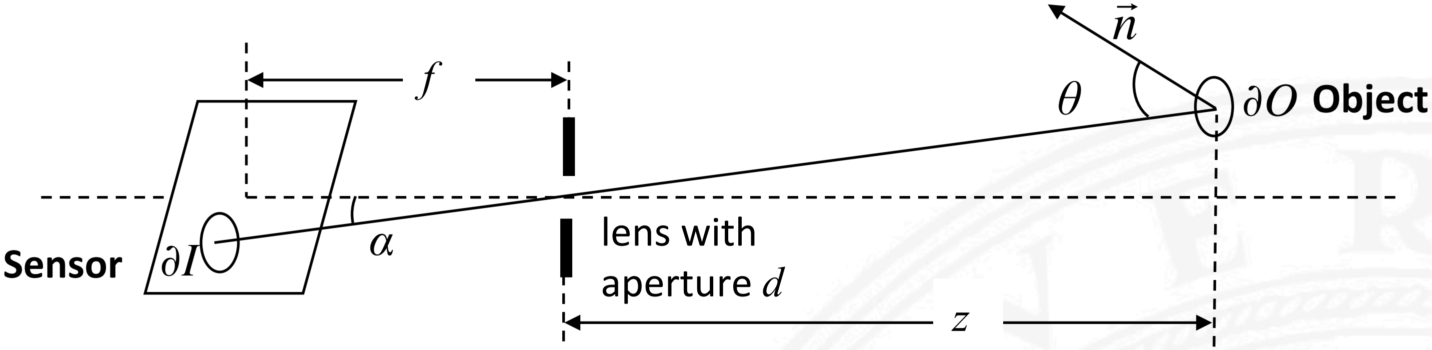
Surface patch produces radiance L,

has spatial angle $\frac{\partial O \cos\theta}{(z/\cos\alpha)^2}$

Spatial angles must be equal: $\frac{\partial O}{\partial I} = \frac{\cos\alpha}{\cos\theta}\frac{z^2}{f^2}$ (Eq. 20-5)

Spatial angle of aperture
for surface patch: $\Omega_L = \frac{\pi}{4}\frac{d^2\cos\alpha}{(z/\cos\alpha)^2} = \frac{\pi}{4}(\frac{d}{z})^2\cos^3\alpha$

Irradiance of Imaging Device (2)



Contribution of surface element to radiant flux Φ at lens:

$$\partial \Phi = L\ \partial O\ \Omega_L \cos\theta = \pi L\ \partial O\ (\frac{d}{z})^2 \frac{\cos^3 \alpha \cos\theta}{4}$$

Irradiation E on image patch:

$$E = \frac{\partial \Phi}{\partial I} = L \frac{\partial O}{\partial I} \frac{\pi}{4} (\frac{d}{z})^2 \cos^3 \alpha \cos\theta$$

With Eq. (20-5) one gets:

$$E = L \frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha$$

Sensor signal depends on span-off angle α of surface element ("vignetting")

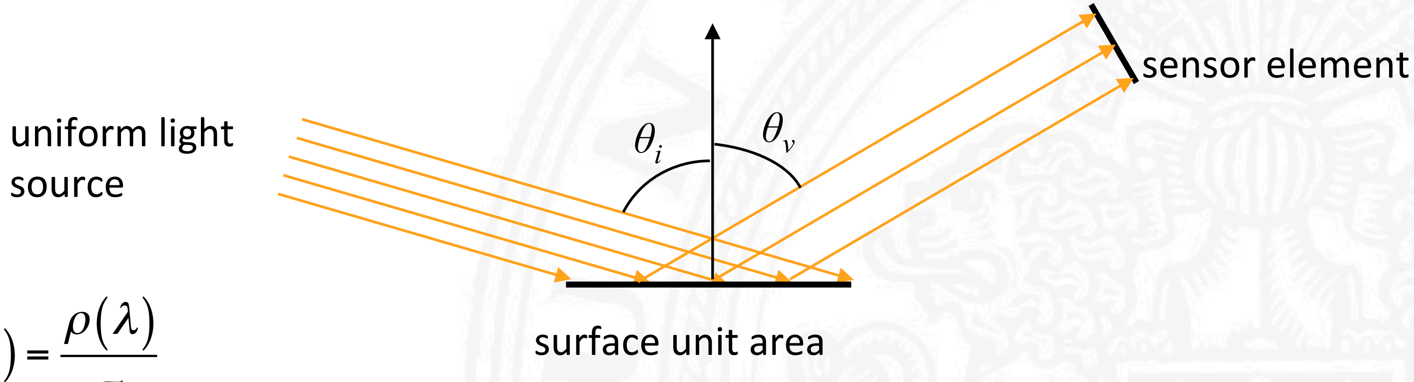
⇒ **off-center pixels in wide-angle images are darker**

Lambertian Surfaces

A **Lambertian surface** is an ideally matte surface which looks equally bright from all viewing directions under uniform or collimated illumination. Its brightness is proportional to the cosine of the illumination angle.

- surface receives energy per unit area ~ $\cos(\theta_i)$
- surface reflects energy ~ $\cos(\theta_v)$ due to matte reflectance properties
- sensor element receives energy from surface area ~ $1/\cos(\theta_v)$

cancel out!



$$r_{Lambert}(\theta_i, \theta_v, \varphi) = \frac{\rho(\lambda)}{\pi}$$

$$\rho(\lambda) = \frac{\int_\Omega L \partial\Omega}{E_i}$$

"albedo" = proportion of incident energy reflected back into half space Ω above surface

Principle of Shape from Shading

See "Shape from Shading" (B.K.P. Horn, M.J. Brooks, eds.), MIT Press 1989

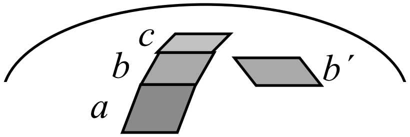
Physical surface properties, surface orientation, illumination and viewing direction determine the greyvalue of a surface patch in a sensor signal.

For a single object surface viewed in one image, greyvalue changes are mainly caused by surface **orientation changes**.

The reconstruction of **arbitrary** surface shapes is not possible because different surface orientations may give rise to identical greyvalues.

Surface shapes may be uniquely reconstructed from shading information if possible surface shapes are constrained by **smoothness assumptions**.

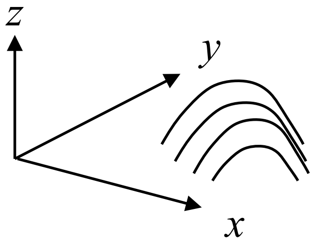
Principle of incremental procedure for surface shape reconstruction:



a:	patch with known orientation
b, c:	neighbouring patches with similar orientations
b':	radical different orientation may not be neighbour of a

Surface Gradients

For 3D reconstruction of surfaces, it is useful to represent reflectance properties as a function of surface orientation.



$z(x, y)$	surface
$p = \delta z / \delta x$	x-component of surface gradient
$q = \delta z / \delta y$	y-component of surface gradient

$\begin{pmatrix} 1 \\ 0 \\ p \end{pmatrix}$ tangent vector in x direction

$\begin{pmatrix} 0 \\ 1 \\ q \end{pmatrix}$ tangent vector in y direction

$\begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix}$ vector in surface normal direction

$\vec{n} = \frac{1}{\sqrt{1 + p^2 + q^2}} \begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix}$ surface normal vector

If the z-axis is chosen to coincide with the viewer direction, we have

$$\cos\theta_v = \frac{1}{\sqrt{1 + p^2 + q^2}} \qquad \cos\theta_i = \frac{1 + p_i p + q_i q}{\sqrt{1 + p^2 + q^2}\,\sqrt{1 + p_i^2 + q_i^2}} \qquad \cos\varphi = \frac{1}{\sqrt{1 + p_i^2 + q_i^2}}$$

The dependency of the BRDF on θ_i, θ_v and φ may be expressed in terms of p and q (with p_i and q_i for the light source direction).

Simplified Image Irradiance Equation

Assume that

- the object has uniform reflecting properties,
- the light sources are distant so that the irradiation is approximately constant and equally oriented,
- the viewer is distant so that the received radiance does not depend on the distance but only on the orientation towards the surface.

With these simplifications the sensor greyvalues depend only on the surface gradient components p and q.

$$E(x,y) = R\big(p(x,y), q(x,y)\big) = R\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right)$$

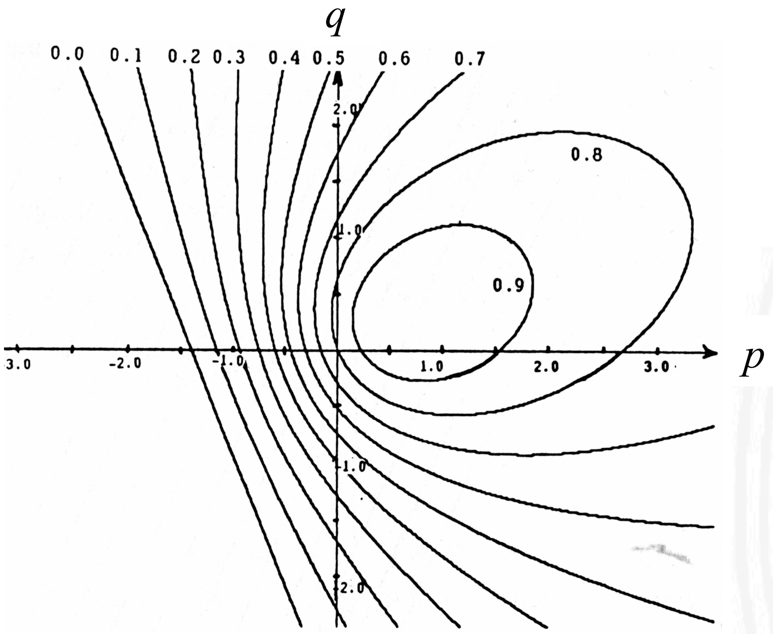
"Simplified Image Irradiance Equation"

$R(p, q)$ is the reflectance function for a particular illumination geometry. $E(x, y)$ is the sensor greyvalue measured at (x, y). Based on this equation and a smoothness constraint, shape-from-shading methods recover surface orientations.

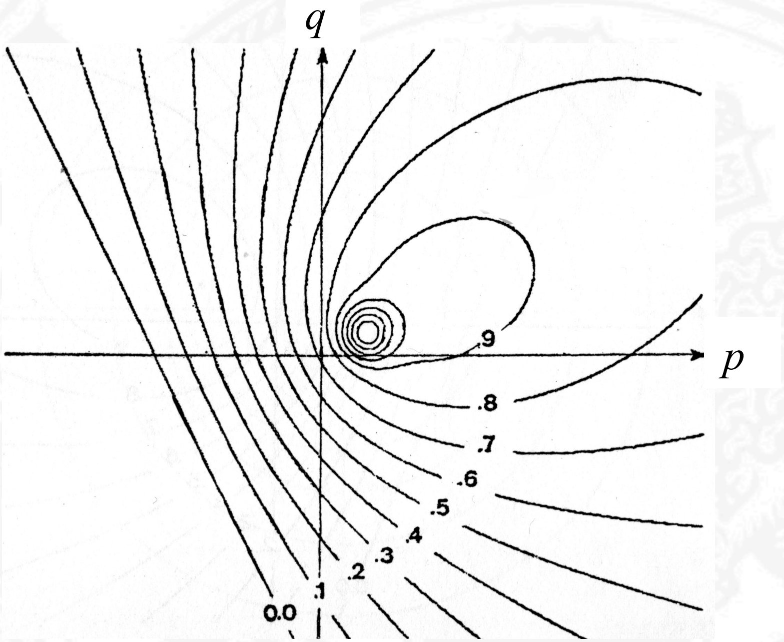
Reflectance Maps

R(p, q) **may be plotted as a reflectance map with iso-brightness contours.**

Reflectance map for Lambertian surface illuminated from $p_i = 0.7$ and $q_i = 0.3$



Reflectance map for matte surface with specular component



Characteristic Strip Method

Given a surface point (x, y, z) with known height z, orientation p and q, and second derivatives $r = z_{xx}$, $s = z_{xy} = z_{yx}$, $t = z_{yy}$, the height $z+\Delta z$ and orientation $p+\Delta p$, $q+\Delta q$ in a neighbourhood $x+\Delta x$, $y+\Delta y$ can be calculated from the image irradiance equation $E(x, y) = R(p, q)$.

- Infinitesimal change of height: $\Delta z = p\,\Delta x + q\,\Delta y$
- Changes of p and q for a step Δx, Δy: $\Delta p = r\,\Delta x + s\,\Delta y$ $\Delta q = s\,\Delta x + t\,\Delta y$
- Differentiation of image irradiance equation w.r.t. x and y gives

  $$E_x = r\,R_p + s\,R_q \quad E_y = s\,R_p + t\,R_q$$

- Choose step $\Delta\xi$ in gradient direction of of the reflectance map ("characteristic strip"):

  $$\Delta x = R_p\,\Delta\xi \quad \Delta y = R_q\,\Delta\xi$$

- For this direction the image irradiance equation can be replaced by

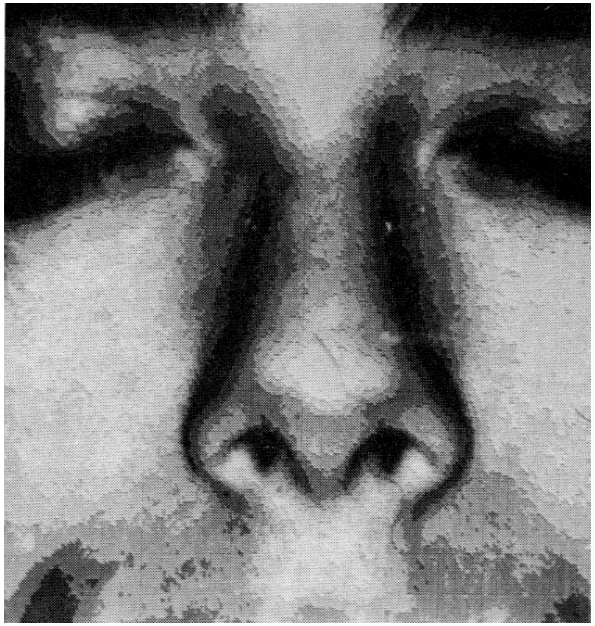
  $$\Delta x/\Delta\xi = R_p \quad \Delta y/\Delta\xi = R_q \quad \Delta z/\Delta\xi = p\,R_p + q\,R_q \quad \Delta p/\Delta\xi = E_x \quad \Delta q/\Delta\xi = E_y$$

Boundary conditions and initial points may be given by

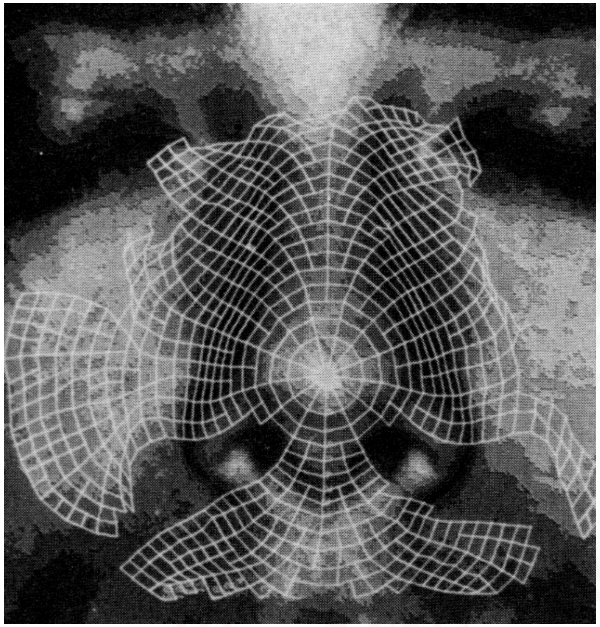
- occluding contours with surface normal perpendicular to viewing direction
- singular points with surface normal towards light source.

Recovery of the Shape of a Nose

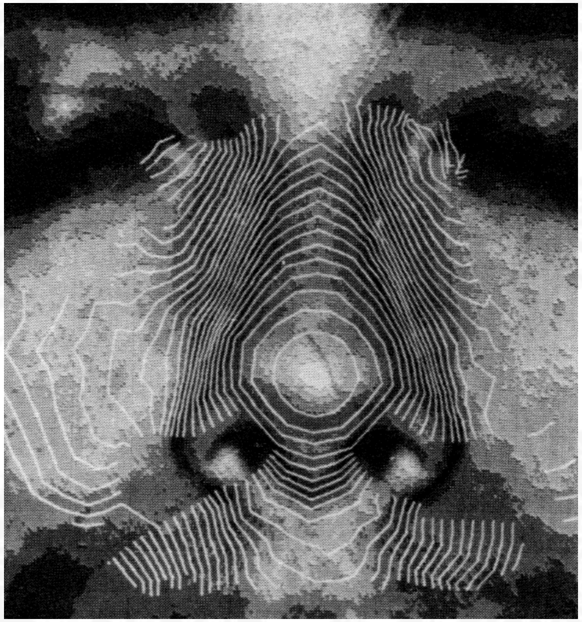
Pictures from B.K.P. Horn "Robot Vision", MIT Press,1986, p. 255



nose with crudely quantized greyvalues



superimposed characteristic curves



superimposed elevations at characteristic curves

Note: Nose has been powdered to provide Lambertian reflectance map!

Shape from Shading by Global Optimization

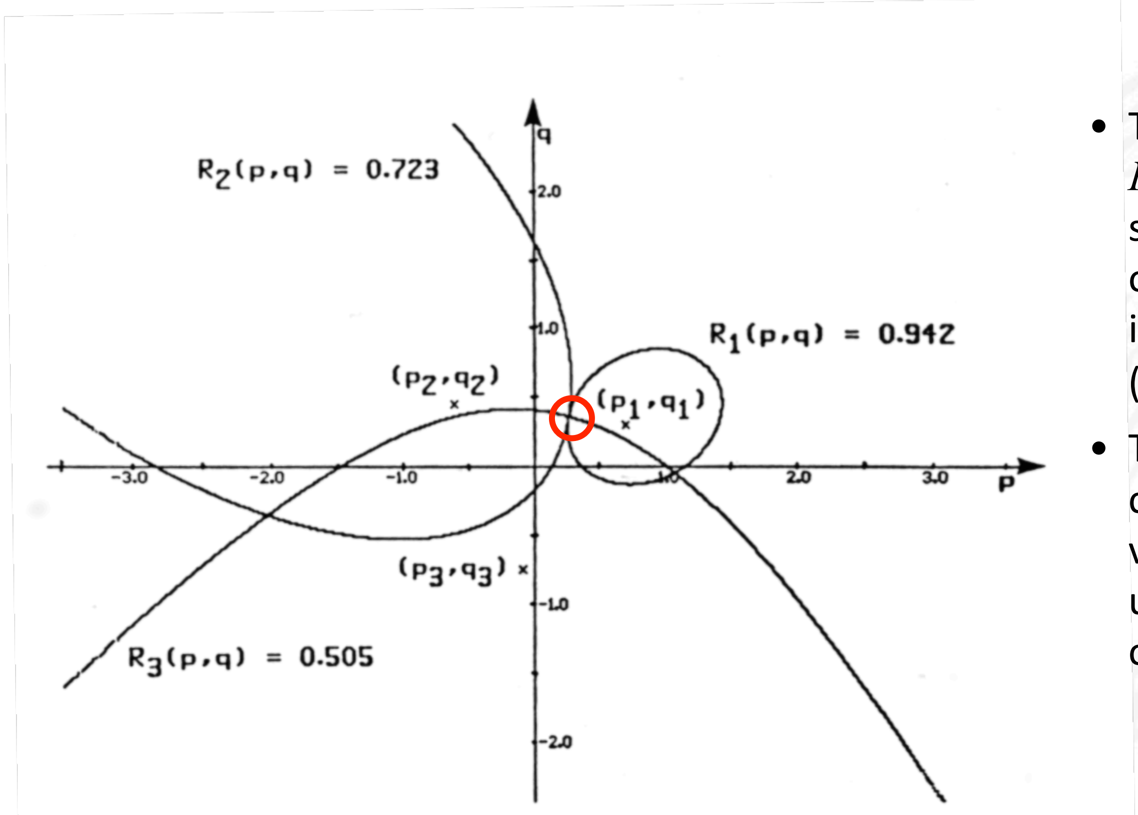
Given a monocular image and a known image irradiance equation, surface orientations are ambiguously constrained. Disambiguation may be achieved by optimizing a global smoothness criterion.

Minimize $$D(x,y)=\underbrace{\left[E(x,y)-R(p,q)\right]^2}_{\text{violation of reflectance constraint}}+\underset{\substack{\uparrow\\ \text{Lagrange multiplier}}}{\lambda}\underbrace{\left[\left(\nabla^2 p\right)^2+\left(\nabla^2 q\right)^2\right]}_{\text{violation of smoothness constraint}}$$

- There exist standard techniques for solving this minimization problem iteratively. In general, the solution may not be unique.
- Due to several uncertain assumptions (illumination, reflectance function, smoothness of surface) solutions may not be reliable.

Principle of Photometric Stereo

In photometric stereo, several images with different known light source orientations are used to uniquely recover 3D orientation of a surface with known reflectance.



From "Shape from Shading“, B.K.P. Horn and M.J. Brooks (eds.), MIT Press 1989

- The reflectance maps $R_1(p, q), R_2(p, q), R_3(p, q)$ specify the possible surface orientations of each pixel in terms of iso-brightness contours ("isophotes").
- The intersection of the isophotes corresponding to the 3 brightness values measured for a pixel (x, y) uniquely determines the surface orientation $(p(x, y), q(x, y))$.

Analytical Solution for Photometric Stereo

For a Lambertian surface:

$$E(x,y) = R(p,q) = \rho \cos\left(\Theta_i\right) = \rho \ \vec{i}^{\,T} \vec{n}$$

$\vec{i}$ = light source direction, $\vec{n}$ = surface normal, ρ = constant

If K images are taken with K different light sources $\underline{i}_k$, $k = 1 \ldots K$, there are K brightness measurements E_k for each image position (x, y):

$$E_k(x,y) = \rho \ \left(\vec{i}_k\right)^T \vec{n}$$

Matrix notation

$$\vec{E}(x,y) = \rho \ L^T \vec{n} \quad \text{with} \quad L = \begin{pmatrix} \left(\vec{i}_1\right)^T \\ \vdots \\ \left(\vec{i}_K\right)^T \end{pmatrix}$$

For K=3, L may be inverted, hence: $\vec{n}(x,y) = \dfrac{L^{-1}\vec{E}(x,y)}{\left\| L^{-1}\vec{E}(x,y) \right\|}$

In general, the pseudo-inverse must be computed: $\vec{n}(x,y) = \dfrac{(L^T L)^{-1} L^T \vec{E}(x,y)}{\left\| (L^T L)^{-1} L^T \vec{E}(x,y) \right\|}$